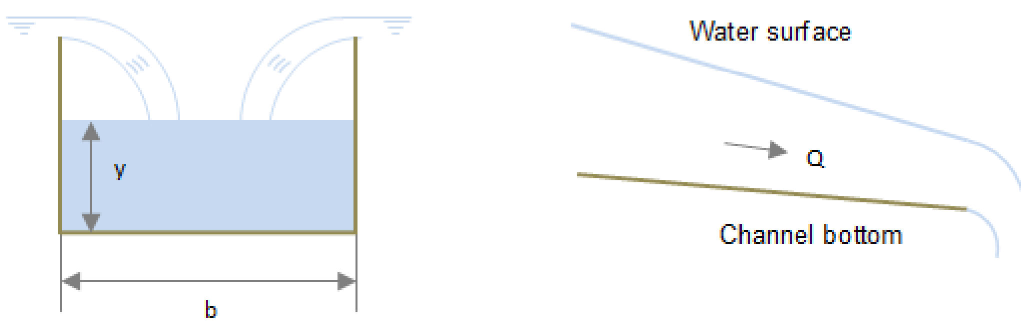
Spatially Varied Open-Channel Flow with Increasing Discharge

There are many applications of spatially varied flow with increasing discharge, including roof gutters and channel spillways. The governing differential equation is

$$\frac{d}{dx}y(x) = \frac{S_0 - S_f - \frac{2 \cdot Q}{g \cdot A^2} \cdot \frac{d}{dx}Q}{1 - Fr^2}$$

where
-S0 is the slope of the channel bed
- Sf is the energy gradient (or the rate at which energy is lost via friction) as given by the Manning formula
- Fr is the Froude number
- Q is the discharge
- x is the distance along the channel
- y is the height of liquid above the channel bed.

Consider a long rectangular channel that ends in an abrupt free fall, with an inflow of water along its length. This application will calcualte the profile of the water surface from the free fall to a specified distance upstream



For a subcritical flow, downstream conditions determine the water surface profile. The water depth reaches the critical height (i.e. the minimum energy height) near the free fall - this is the boundary condition on the differential equation.

Width, slope and friction of rectangular channel $b := 4$ $S_0 := 0.0004$ $n := 0.013$

Gravity $g := 9.81$

Distance from the freefall to the terminal point of analysis $L := 2000$

Discharge (note the dependency on x) $Q := 0.02 \cdot L - 0.02 \cdot x$

Cross-sectional area of flow $A := b \cdot y(x)$

Wetted perimeter $P := b + 2 \cdot y(x)$

Hydraulic radius $H := \frac{A}{P} = \frac{4 \cdot y(x)}{4 + 2 \cdot y(x)}$

Froude number $Fr := \sqrt{\frac{Q^2 \cdot b}{g \cdot A^3}} = 0.080 \cdot \sqrt{\frac{(40.000 - 0.020 \cdot x)^2}{y(x)^3}}$

Slope of the energy gradient from the Manning equation $S_f := \left(\frac{n \cdot Q}{A \cdot H^{\frac{2}{3}}}\right)^2 = \frac{0.010 \cdot (0.520 - 2.600 \times 10^{-4} \cdot x)^2}{y(x)^2 \cdot \left(\frac{y(x)}{4 + 2 \cdot y(x)}\right)^{4/3}}$

Hence the differential equation is $de := \frac{d}{dx} y(x) = -\frac{S_0 - S_f - \frac{2 \cdot Q}{g \cdot A^2} \cdot \frac{d}{dx} Q}{1 - Fr^2}$

$$de = \frac{d}{dx}\, y(x) = -\frac{4.000 \times 10^{-4} - \frac{0.010 \cdot (0.520 - 2.600 \times 10^{-4} \cdot x)^2}{y(x)^2 \cdot \left(\frac{y(x)}{4 + 2 \cdot y(x)}\right)^{4/3}} + \frac{1.274 \times 10^{-4} \cdot (80.000 - 0.040 \cdot x)}{y(x)^2}}{1 - \frac{0.006 \cdot (40.000 - 0.020 \cdot x)^2}{y(x)^3}}$$

Critical flow depth for a rectangular channel. This is the downstream boundary condition at the freefall $y_c := \left(\frac{\text{eval}(Q, x = 0)^2}{b^2 \cdot g}\right)^{1/3} = 2.168$

Numerical solution of the differential equation

```
res := dsolve({de, y(L) = y_c}, numeric,
output = listprocedure)

y := subs(res, y(x))
```

Plot of water depth along channel

```
plot(y(x), x = 0..L, gridlines, color = "DarkBlue", thickness = 4,
title = "Water Surface in a Rectangular Channel", titlefont = [Arial, 12]) =
```