



# Maple Conference 2021

November 2 - 5, 2021

**VIRTUAL**

Event Guide

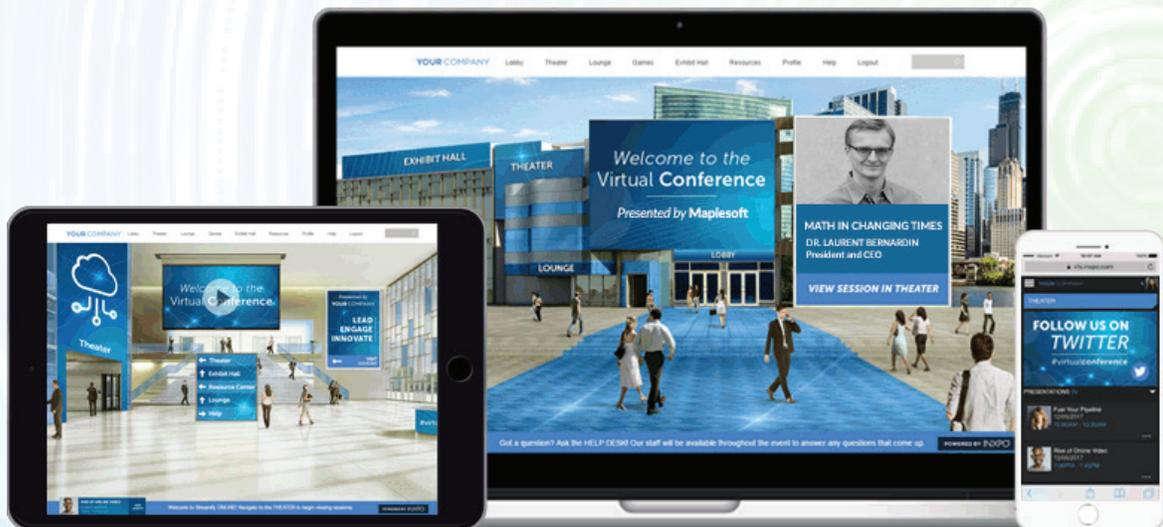
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# Welcome to the 2021 Maple Conference!

Welcome to the 2021 Maple Conference! This conference is dedicated to exploring different aspects of the math software Maple, including Maple's impact on education, new symbolic computation algorithms and techniques, and the wide range of Maple applications. You will have the opportunity to learn about the latest research, share experiences, and interact with Maple developers.



## Where to Go

The Maple Conference is presented online in an **interactive virtual environment** complete with a presentation theater, networking lounge and other virtual facilities.

- ▶ On arrival, go to **Profile** to fill in your virtual name tag. This makes it easier for your fellow attendees to find you in the lounge. Come back later to see what badges you've earned in the Maple Badge Game
- ▶ Go to the **Theater** to watch recorded presentations and attend live events, including the keynotes, discussion panels, and Q&A sessions
- ▶ In the **Lounge**, you can join a social Zoom room or initiate a text or video chat with another attendee.
- ▶ Enjoy some mathematical art in the **Art Gallery**, and vote for your favorite!
- ▶ In the **Resource Library**, access a variety of written and visual materials about Maplesoft products.
- ▶ Use the **Help** menu for support questions related to the virtual event environment.
- ▶ Visit the **Information Desk** in the **Lobby** to ask questions about the conference, to request a meeting with your Maplesoft Account Manager, or ask about anything related to Maplesoft.

## Anti-Harassment Policy

The Maple Conference is dedicated to providing a harassment-free conference experience for everyone. We do not tolerate harassment of conference participants in any form. Conference participants violating these rules may be expelled from the conference at the discretion of the conference organizers. Participants asked to stop any harassing behavior are expected to comply immediately. You can report a violation of this policy to any Maplesoft staff or by emailing [mapleconference@maplesoft.com](mailto:mapleconference@maplesoft.com).

## Territorial Acknowledgment

Maplesoft's headquarters is located in Waterloo, Ontario, Canada. We acknowledge that Waterloo Region, including the three cities and four townships, is located on the traditional territory of the Haudenosaunee, Anishnaabe and Neutral People. We acknowledge the enduring presence of the Indigenous people with whom we share this land, their achievements and their contributions to our community. We offer this acknowledgement as an act of reconciliation between Indigenous and non-Indigenous peoples of Canada.

## Overview Schedule

All times are given in Eastern Daylight Time (EDT).

	Mon. Nov. 1	Tues. Nov. 2	Wed. Nov. 3	Thurs. Nov. 4	Fri. Nov. 5				
8am-9am		Networking	Networking	Networking	Networking				
9am-10am	<b>Workshop</b> Advanced Problem Solving with Regular Chains	<b>Keynote</b> <b>Dr. Veselin Jungic</b> Two-Eyed Seeing: Mathematics and Indigenous Traditions and Cultures	<b>Keynote</b> <b>Dr. Laurent Bernardin</b> Math in Changing Times	<b>Discussion Panel</b> Another Famous Unsolved Problem: Improving Diversity in STEM	<b>Keynote</b> <b>Dr. Evelyne Hubert</b> An Integral View on Dimensional Analysis: Scaling Invariants for Parameter Reductions in Dynamical Systems				
10am-10:15am		Break	Break	Break	Break				
10:15am-11:05am		<b>Q&amp;A</b> - Maple in Education	<b>Q&amp;A</b> - Applications of Maple	<b>Q&amp;A</b> - Maple in Education	<b>Q&amp;A</b> - Applications of Maple	<b>Q&amp;A</b> - Maple in Education	<b>Q&amp;A</b> - Algorithms and Software	<b>Q&amp;A</b> - Maple in Mathematics	<b>Q&amp;A</b> - Maple in Mathematics
11:05am-11:20am		Break	Break	Break	Break				
11:20am-12:10pm		<b>Q&amp;A</b> - Applications of Maple	<b>Q&amp;A</b> - Algorithms and Software	<b>Discussion Panel</b> Meet the Developers	<b>Q&amp;A</b> - Applications of Maple	<b>Q&amp;A</b> - Algorithms and Software	<b>Q&amp;A</b> - Maple in Mathematics	<b>Q&amp;A</b> - Algorithms and Software	
12:10pm-1pm			Networking and Maple Ambassador Meeting	Networking	Art Gallery Event and Networking				
1pm-4pm	<b>Workshop</b> Maple Programming: Beyond the Basics								



## Maple Badge Game

There's plenty to do at this year's Maple Conference, with all the live and recorded presentations to watch, discussion panels to attend, questions to ask, and so much more. How could it get any better?!

With a chance to **win fabulous prizes**, of course!



Actual prizes may vary

All Maple Conference attendees are automatically enrolled to participate in our Maple Badge Game. Badges are awarded for completing specific challenges and have an allotted point value. Once you complete a challenge, you will automatically earn that badge's point value and the virtual badge will be displayed in your profile. The highest point earners will be entered into a draw for some awesome Maple swag!

Look in your Profile to see which badges you've earned, and check out the Leaderboard in the Prize Center to see the latest standings.

## Badge List



### PROFILE MASTER

Complete the attendee profile, tour the virtual venue, and join the virtual conference during the live event (Nov. 2- Nov. 5, 8am-1pm EDT).

**Earn 50 points**



### SESSION SEEKER

Attend a Q&A session.

**Earn 150 points**



### LEADERSHIP

Attend all three keynote presentations.

**Earn 150 points**



### SOCIALIZER

Join the conversation in the networking room in the Lounge. **Earn 150 points**



### EXPLORER

Watch four recorded presentations from any one stream and participate in a live Q&A.

**Earn 125 points**



### INNOVATOR

Watch two recorded presentations from two different streams, and participate in a live Q&A.

**Earn 125 points**



### DIGITAL PIONEER

Attend a Discussion Panel. **Earn 100 points**



### GRAB THE MONET AND LET'S GOGH!

Visit the Art Gallery and vote for your favorite piece. **Earn 100 points**



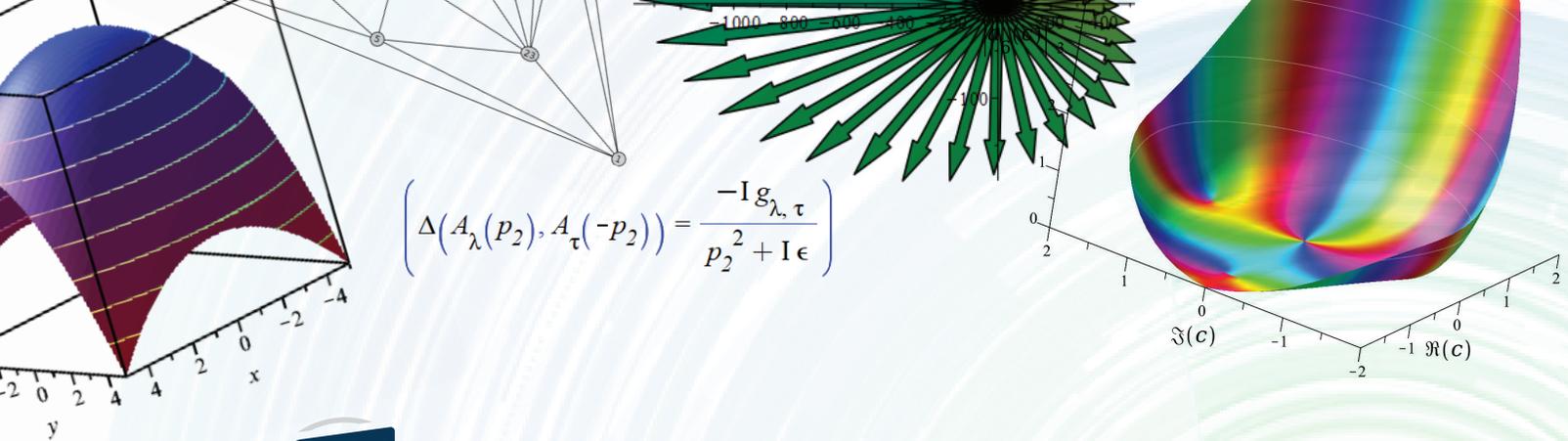
### CONNECTOR

Initiate a one-on-one text or video chat with another attendee. **Earn 50 points**



### MASTER MAPLE ENTHUSIAST

Watch four recorded presentations and attend four live sessions (Q&As, Keynotes, and Discussion Panels). **Earn 200 points**



$$\left( \Delta(A_\lambda(p_2), A_\tau(-p_2)) = \frac{-I g_{\lambda, \tau}}{p_2^2 + I \epsilon} \right)$$



# Maple™ Transactions

## A new open access scholarly online journal

The purpose of Maple Transactions is the dissemination of excellent expositions on topics of interest to the Maple Community

- Focused on computer-assisted mathematics for research, applications, and education
- Refereed articles
- Opinion pieces, puzzles, video expositions, programming column, and more
- Use of Maple not a prerequisite for publication
- Open access, online journal - free for readers and contributors

[mapletransactions.org](http://mapletransactions.org)

## Call for Papers - Due Nov. 15

All presenters and invited speakers at the Maple Conference 2021 are invited to submit a full paper on the work they present at the conference. These papers will undergo peer-review, and if accepted, will appear in a special Maple Conference 2021 Proceedings issue of Maple Transactions.

*A meeting of the Associate Editors of Maple Transactions will take place during the conference, on Tuesday at 1:00 pm. Please check your email for instructions on how to join.*

# Detailed Daily Schedule

## Monday

Two optional add-on workshops are available to attendees of the conference, which take place the day before the conference begins. There is no cost to attend, but registration is required. These workshops are offered as an option during conference registration.

### Maple Programming: Beyond the Basics

**1:00-4:00 pm**

Instructors: Matt Calder and Paulina Chin, Maplesoft

Are you already familiar with the basics of the Maple programming language, and looking to take your skills to the next level? If so, this course is for you! In this hands-on workshop, we will introduce you to tools and techniques that will help you write more effective and powerful Maple code. Topics will include: building larger programs and applications, sharing your code with other Maple users, writing more efficient code, and using tools such as the debugger to aid in programming. Attendees should have at least a basic familiarity with Maple programming before attending this workshop.

### Advanced Problem Solving with Regular Chains

**9:00 am -12:00 pm**

Instructor: Marc Moreno Maza, Western University

Regular chains were originally introduced to solve systems of polynomial equations over the complex numbers, or, to be more technical, over algebraically closed fields. One algorithm, called "Triangularize" decomposes the zero set of a polynomial system into geometrically meaningful components, each of them encoded by a regular chain.

Over the past twenty years, regular chains have proven to be versatile as they can also be used to solve systems of polynomial constraints over the real numbers, possibly in the presence of infinitely many solutions or parameters. The algorithm "Triangularize" could be adapted to perform cylindrical algebraically decomposition (CAD) in a novel way (proceeding incrementally, one constraint after another), yielding quantifier elimination (QE) based on regular chains.

The RegularChains package in Maple provides a collection of tools for dealing with systems of polynomial equations, inequations and inequalities. These tools include the functionalities mentioned above, as well as more specialized functionalities, such as counting solutions without computing them, performing set theoretical operations on constructible set or semi-algebraic sets, etc.

This tutorial will start with a tour of these functionalities before diving into some of the RegularChains sub-packages and related libraries, dedicated to applications. With the latter, we will cover parametric linear systems, computations of limit points, real branches of space curves, intersection multiplicities and more.

## Tuesday | At a Glance

Time	Event	Location
8 am – 9 am	Networking	Lounge
9 am – 10 am	<b>Keynote - Dr. Veselin Jungic</b> Two-Eyed Seeing: Mathematics and Indigenous Traditions and Cultures	Theater
10 am – 10:15 am	Break	

10:15 am – 11:05 am	<p style="text-align: center;"><b>Track 1</b> <b>Q&amp;A – Maple in Education</b></p> <p><b>Tomas Recio and M. Pilar Vélez</b> Niagara Falls and the origins of Computer Algebra</p> <p><b>Jürgen Gerhard</b> Maple 2021 Gems You May Have Missed</p> <p><b>Zoltán Kovács, Rafael Losada and Tomas Recio</b> Discovering geometric inequalities: the concourse of GeoGebra Discovery, Dynamic coloring and Maple tools</p> <p><b>Robert Lopez</b> Does Jordan's Ghost Haunt <math>x' = Ax</math> When A is Defective?</p>	<p style="text-align: center;"><b>Track 2</b> <b>Q&amp;A – Applications of Maple</b></p> <p><b>Iliia Ilmer, Alexey Ovchinnikov and Gleb Pogudin</b> MapleCloud in Symbolic Algebra Research: Hosting Software for Structural Identifiability</p> <p><b>Maxim Sakharov and Andrey Sukhoruchkin</b> Calculation of the Dose Coefficient for Radioactive Aerosol of an Arbitrary Disperse Composition</p> <p><b>Daulet Nurakhmetov, Serik Jumabayev, Almir Aniyarov and Rinat Kussainov</b> On quality properties of eigenvalues of EulerBernoulli beams under axial loads</p> <p><b>Vladimir A. Filin and Valentina A. Yurova</b> Symbolic Analysis of Linear Amplifiers with Multi-Loop Feedbacks in Interacting Programs Maple and FASTMEAN</p>	Theater
11:05 am – 11:20 am	Break		
11:20 am – 12:10 pm	<p style="text-align: center;"><b>Track 1</b> <b>Q&amp;A – Applications of Maple</b></p> <p><b>Thomas Schramm</b> Local and global properties of the gravitational lens effect with special consideration of the gravitational lens effect with star perturbation</p> <p><b>Kays Haddad and Edgardo Cheb-Terrab</b> FeynmanIntegral: Symbolic Feynman Integral Evaluation in Maple</p> <p><b>Davide Polvara and Edgardo Cheb-Terrab</b> Physics: -FeynmanDiagrams: A Maple Package for the Computation of Scattering Amplitudes</p> <p><b>William Fajardo</b> Maple Implementation of the Skew PBW Extensions</p>	<p style="text-align: center;"><b>Track 2</b> <b>Q&amp;A – Algorithms and Software</b></p> <p><b>Jürgen Gerhard, Marc Moreno Maza and Ryan Sandford</b> Computing Intersection Multiplicities with the RegularChains Library</p> <p><b>Matt Calder, Juan Pablo Gonzalez Trochez, Marc Moreno Maza and Erik Postma</b> A Maple implementation of a probabilistic algorithm for computing the common zeros of a polynomial and a regular chain</p> <p><b>Laureano Gonzalez-Vega</b> Using Maple to Study Bohemian Correlation Matrices</p> <p><b>Sergey Yurkevich</b> The Art of Educated Algorithmic Guessing</p>	Theater
12:10 pm – 1 pm	Networking		Lounge
12:10 pm – 1 pm	Maple Ambassador Meeting (by invitation)		Theater

## Tuesday | Detailed Schedule

Time	Event	Location
8:00 am	Networking	Lounge
9:00 am	<b>Keynote</b>	Theater



**Dr. Veselin Jungic, Simon Fraser University**

### Two-Eyed Seeing: Mathematics and Indigenous Traditions and Cultures

Elder Albert Marshal of the Mi'kmaw Nation describes "two-eyed seeing" as the ability to see with the strength of Indigenous knowledge from one eye while seeing with the strength of Western knowledge from the other. This dual perspective can be applied to many aspects of life, including mathematics.

In this presentation, I will explore the concept of "two-eyed seeing" and the field of ethnomathematics, the study of the relationship between mathematics and culture first introduced by Brazilian educator and mathematician Ubiratan D'Ambrosio. I will address some of the dynamics between these two concepts and illustrate them with several examples. These examples will include a brief analysis of the geometry evident in a traditional Haida Nation hat, as well as the work of contemporary Salish artist Dylan Thomas.

In addition, I will discuss a project that used mathematical modeling of a traditional Tla'amin Nation stone fish trap to communicate cultural, engineering, environmental, and mathematical ideas. This project was a collaboration with the Tla'amin Nation and the Callysto Program, an online education tool that helps students in elementary and high school learn about and apply data science skills.

10:00 am	Break	
10:15 am	<b>Track 1: Q&amp;A - Maple in Education</b>	Theater



**Tomas Recio (Universidad Antonio de Nebrija) and M. Pilar Vélez (Universidad Antonio de Nebrija)**

### Niagara Falls and the origins of Computer Algebra

It is well known and documented the specific relation between Charles Babbage and Computer Algebra, that is, not just considering the work of Babbage towards the development of a kind of predecessor of a computer, but regarding his --less popular, but very appreciated by himself-- aim towards machines allowing the exact manipulation of algebraic formulae. Describing and designing machines that perform, exactly --in its abstract design, although not necessarily in reality-- algebraic operations, was a very relevant challenge along the whole XIX century, and up to the present days (cf. Thurston's conjecture on the universality of linkages, and its solution by Kapovich and Millson in 2002), a history that can be appreciated behind some computer algebra concepts. Besides dealing with Babbage's inventions and their connection with the race towards designing algebraic mechanisms, the other ingredient we would like to present in our talk is related to the proposal of L. Torres-Quevedo to the Académie des Sciences, Paris, in 1895, reported by M. Deprez, H. Poincaré and P. Appell (an excellent trio of reviewers!). The report was published in the *Compte Rendus* of the Academy of Sciences, February 19, 1900, highlighting that "...M. Torres a donné une solution théorique générale et complète du problème de la construction des relations algébriques et transcendentes par des machines... (...M. Torres has presented a complete and general theoretical solution to the problem of the construction of algebraic and transcendental relations by machines...)". M. Torres, that is, Monsieur Torres, is the Spanish scientist and engineer Leonardo Torres-Quevedo ([https://en.wikipedia.org/wiki/Leonardo\\_Torres\\_y\\_Quevedo](https://en.wikipedia.org/wiki/Leonardo_Torres_y_Quevedo)), quite popular for some of his devices (such as the one still operating in Niagara Falls, cf. [https://en.wikipedia.org/wiki/Whirlpool\\_Aero\\_Car](https://en.wikipedia.org/wiki/Whirlpool_Aero_Car)), but little known concerning his early work towards the development of what he called "algebraic machines", aiming to contribute to the advance of Babbage's theory.

Thus, our contribution aims, essentially, to highlight the triple connection: Origins of Computer Algebra— Babbage— Algebraic Machines—Torres-Quevedo... and, as a consequence, to convene the Computer Algebra community towards some interesting historical issues, still with a potential scientific impact in present times!



In this work, we present a web-based program that allows us to quickly assess identifiability of individual parameters and initial conditions by a Monte-Carlo randomized algorithm with a user-prescribed probability as well as deterministically find all parameter functions identifiable from single or multiple experiments. The program is hosted on MapleCloud and is implemented in the Maple programming language. The underlying algorithms for both individual identifiability and identifiability of combinations rely on computing Gröbner basis using F4 algorithm and characteristic sets via Rosenfeld-Gröbner algorithm of the `DifferentialAlgebra` package. To mitigate computational bottlenecks arising from Rosenfeld-Gröbner implementation in Maple, we will show how one can bypass finding identifiable combinations for certain cases at the cost of making an otherwise deterministic result a probabilistic one. We will also discuss usage of the `DifferentialThomas` package as a replacement to `DifferentialAlgebra`. We will additionally discuss some empirical observations regarding Gröbner basis computation that allow us to gain more speedup in the application.

The structural identifiability toolbox is freely available online at <https://maple.cloud/app/6509768948056064/>



**Maxim Sakharov (Bauman Moscow State Technical University) and Andrey Sukhoruchkin (National Research Centre "Kurchatov Institute")**

### **Calculation of the Dose Coefficient for Radioactive Aerosol of an Arbitrary Disperse Composition**

A dose coefficient for an aerosol is the value of the human's irradiation dose caused by radionuclide inhalation intake in the quantity of one activity unit, Sv/Bq. This is one of the key metrics for radiological safety and radiation medicine applications. A dose coefficient depends on the disperse composition of an aerosol. In other words, it depends on the distribution of radionuclide activity over the aerosol's particles of different diameters. In the International Commission on Radiological Protection (ICRP) database each presented value of aerosol's dose coefficient corresponds to one out of ten standard functions of logarithmic-normal distribution with given parameters. Empirical measurements of activity distributions can differ from these standard functions to some extent.

This work proposes a method for approximation of an arbitrary distribution function of aerosol activity over its particles' diameters based on the weighted arithmetical sum of several most appropriate standard functions. The least square method was used to determine the weights of functions with a use of `NonlinearFit` function in Maple software. The nonlinear approximation procedure was used in order to obtain only positive weights that would correspond to physical properties of the method's justification. Once the weights are determined, a dose coefficient of the aerosol under investigation is calculated using the additivity property of dose values: each weight is multiplied by a known dose coefficient value from the ICRP database, and the obtained results are summed up.

Maple software helps to control the approximation quality by monitoring different types of residuals. In order to select the most appropriate standard functions as well as the methods of visual control and presentation of the obtained results an algorithm was developed in Maple for building distribution plots in logarithmic probability scale. Such a plot type is often used for analyzing radio contamination of aerial environment.



**Daulet Nurakhmetov (Institute of mathematics and mathematical modelling), Serik Jumabayev (Institute of Management, Academy of Public Administration under the President of the Republic of Kazakhstan), Almir Aniyarov (Institute of mathematics and mathematical modelling) and Rinat Kussainov (Institute of mathematics and mathematical modelling)**

### **On quality properties of eigenvalues of EulerBernoulli beams under axial loads**

We consider a spectral problem of the uniform and non-uniform Euler-Bernoulli beams on the Winkler foundations with various boundary conditions under axial loads. The novelty of the research is in the consideration of the models with an arbitrary variable coefficient of foundation and a constant coefficient of axial load. Qualitative results influence the symmetry of the coefficient of the foundation on the spectral properties were obtained in [1]. Similar results of [1] for beams under axial loads without foundation were shown in [2].

Spectral problems with different boundary conditions under axial loads for Euler-Bernoulli beam equation were investigated in connection with practical applications in [2-4] without foundation and in [4] with constant foundation. Numerical calculations were carried out in the Maple computer mathematics system.

Funding: This work was financially supported by the Ministry of Education and Science of the Republic of Kazakhstan (project AP08052239)

#### References

1. Nurakhmetov D. Symmetric properties of eigenvalues and eigenfunctions of uniform beams / D. Nurakhmetov, S. Jumabayev, A. Aniyarov, R. Kussainov // *Symmetry*. - 2020. - 12 (2097). - P. 1-13.
2. Valle J. Closed-form equation for natural frequencies of beams under full range of axial loads modeled with a spring-mass system / J. Valle, D. Fernandez, J. Madrenas // *International Journal of Mechanical Sciences*. - 2019. - 153-154. - P. 380-390.
3. Bokaian A. Natural frequencies of beams under compressive axial loads // *Journal of Sound and Vibration*. - 1988. - 126:1. - P. 49-65.
4. Bokaian A. Natural frequencies of beams under tensile axial loads // *Journal of Sound and Vibration*. - 1990. - 142:3. - P. 481-498



**Vladimir A. Filin** (The Bonch-Bruевич Saint-Petersburg State University of Telecommunications) and **Valentina A. Yurova** (The Bonch-Bruевич Saint-Petersburg State University of Telecommunications)

**Symbolic Analysis of Linear Amplifiers with Multi-Loop Feedbacks in Interacting Programs Maple and FASTMEAN**

A technique of interaction of computer programs for symbolic analysis of complex electronic circuits with amplifying elements is proposed. The FASTMEAN simulation program, used in the universities of telecommunications in Russia, has a symbolic analysis module and is capable of generating analytical expressions for Laplace images of a circuit determinant, currents and voltages in complex electronic circuits. However, the obtained expressions have a nested (folded) structure, which makes it difficult to analyze the influence of elements on the properties of a circuit with amplifiers and feedbacks, in particular on its stability. It is proposed to transfer the obtained expressions to the Maple program for their structural transformation and mathematical processing.

Amplifiers with local, common and crossed feedbacks are considered. The analysis of such circuits in Maple shows that an expression for the circuit determinant in the form of the products of the loop gain functions is a sign of the presence of several feedback loops in the circuit.

11:05 am	Break	
11:20 am	<b>Track 1: Applications of Maple</b>	Theater



**Thomas Schramm** (HafenCity University Hamburg)

**Local and global properties of the gravitational lens effect with special consideration of the gravitational lens effect with star perturbation**

Since the late 1970s, gravitational lensing became an important tool in astrophysics, taking advantage of the lens-like bending of light by masses such as planets, stars, galaxies, or clusters of them to determine their properties or even their existence. At that time and later in the 80s, the group at the Hamburg observatory around Sjur Refsdal developed many techniques that are still in use to understand and apply the effect. Although the effect is a consequence of Einstein's general theory of relativity, the equations used to describe the effects of masses on light rays are relatively simple. However, in order to answer questions about what a light source looks like through a special lens, or whether there might be multiple images of a light source, the math got quite complicated and the problems were largely solved numerically.

In this article we show, for an important special case of a star in a galaxy as a lens, that the problems of differential geometry that arise can be treated algebraically by a computer algebra system such as Maple and lead to elegant solutions that are generally applicable to mappings from the plane onto the plane.



**Kays Haddad** (Niels Bohr Institute) and **Edgardo Cheb-Terrab** (Maplesoft)

**FeynmanIntegral1 : Symbolic Feynman Integral Evaluation in Maple**

Building on the calculation of S-matrix integrands by the Maple FeynmanDiagrams package, the FeynmanIntegral package performs the computation of Feynman integrals in any spacetime dimension using dimensional regularization. The package includes commands to parametrize integrals using either Feynman or alpha (Schwinger) parametrization; to express one-loop tensor integrals in terms of scalar integrals using the Passarino-Veltman reduction; to compute loop integrals exactly in the dimensional regularization parameter epsilon, or to expand in this parameter; and to perform integrations over Feynman parameters. We demonstrate this functionality.



**Davide Polvara** (Durham University) and **Edgardo Cheb-Terrab** (Maplesoft)

**Physics : -FeynmanDiagrams : A Maple Package for the Computation of Scattering Amplitudes**

In theoretical physics the collisions between elementary particles are ruled by probabilities; this means that if we hit a pair of protons beams in a collider we cannot know the exact result of such a collision but only the probabilities for the different outcomes. During the talk it will be discussed a package in the Maple environment for the generation of such probabilities, that can be obtained by summing over different graphs, known as Feynman diagrams. The different steps, from the definition of the particle model till the generation of the diagrams and associated probabilities, will be covered.

During the talk it will be shown how to generate all the Feynman diagrams contributing to a certain scattering process providing many different functionalities such as the possibility of input user defined interaction Lagrangians, select arbitrary number of loops and removing one particle reducible graphs. This is done maintaining a simple interface so to make the program useful not only for research purpose but also as a pedagogical device in master classes of particle physics.



**William Fajardo** (Universidad Nacional de Colombia)

### Maple Implementation of the Skew PBW Extensions

In this talk we will present the SPBWE library developed in Maple with which it is possible to define and make calculations with skew Poincaré-Birkhoff-Witt extensions (skew PBW extensions for short). These extensions were introduced with the aim of generalizing the PBW extensions defined by Bell and Goodearl, the skew polynomial rings or Ore extensions of injective type introduced by Ore and another noncommutative rings appearing in several contexts. The fundamental importance of skew PBW extensions is that the coefficients do not necessarily commute with the variables, and these coefficients are not necessarily elements of fields, so these objects are different in these two aspects with respect to G-algebras. Many rings and algebras coming from mathematical physics can be described as skew PBW extensions, among of the most remarkable examples are: The usual polynomial ring, the enveloping algebra of a finite dimensional Lie algebra, the Weyl algebra, the additive analogue of the Weyl algebra, diverse rings of operators, Ore algebras of bijective type, some diffusion algebras, the Woronowicz algebra, the dispin algebra, diverse quantum algebras, the q-Heisenberg algebra, the Hayashi algebra, the Witten's deformation, some quadratic algebras in three variables among others. Ring and homological properties of skew PBW extensions have been extensively studied by various authors. In fact, a book has recently been published containing research results on ring theory, homological and computational properties of these extensions. There are no packages other than the SPBWE library that implement general versions of skew PBW extensions and can perform effective calculations with these. The implementation of the SPBWE library allows to make effective some homological applications with skew PBW extensions, moreover it allows to develop new homological applications of Groebner basis theory.

During the development of the talk we will describe the various packages that make up the SPBWE library and illustrate by a demo some of their applications in homological algebra, in particular the calculation of Groebner bases for ideals and modules on bijective skew PBW extensions. The variety of substructures type skew PBW extensions that can be defined is wide and may not only be based on algebra structures but also rings not necessarily algebras. we can conclude that SPBWE library is a very useful not only for investigating constructively homological properties of many algebraic structures that can be described as skew PBW extensions, but also for many eventual applications of them.

11:20 am

**Track 2: Q&A - Algorithms and Software**

Theater



**Jürgen Gerhard** (Maplesoft), **Marc Moreno Maza** (University of Western Ontario) and **Ryan Sandford** (University of Western Ontario)

### Computing Intersection Multiplicities with the RegularChains Library

When an algebraic curve or surface has a singular point, local approximation at that point by a linear space (namely a tangent space) is not possible. Consequently, other techniques must be used instead such as computing tangent cones and intersection multiplicities. In Maple 2021, the support for those computations in the sub-package AlgebraicGeometryTools of the RegularChains library is essentially limited to the case of planar curves, based on the ideas of William Fulton, from his textbook "Algebraic Curves". Those ideas replace the computation of standard bases (or Groebner bases) by the manipulation of regular sequences and regular chains.

In this paper, we report on new developments (features and efficiency improvements) for the sub-package AlgebraicGeometryTools. First, the range of input systems for which intersection multiplicities (at an arbitrary point) can be computed has increased substantially. Indeed, in the earlier version of the sub-package, input systems in dimension higher than two were handled by a reduction to the planar case requiring hypotheses that do not hold generically. In the latest version of the sub-package, this reduction is replaced by an adaptation of Fulton's techniques to higher dimension. While some corner cases still resist, many more input systems from the literature can now be processed.

Second, when both versions (the one in Maple 2021 and the latest one) both succeed in computing the intersection multiplicity of an input polynomial system at some point, then latest version does it faster, typically by one or two orders of magnitude. One key reason for those improvements is algorithmic optimization, in particular the use of criteria which allow us to skip long series of recursive calls in the main algorithm.



**Matt Calder** (Maplesoft), **Juan Pablo Gonzalez Trochez**, **Marc Moreno Maza** (University of Western Ontario) and **Erik Postma** (Maplesoft)

**A Maple implementation of a probabilistic algorithm for computing the common zeros of a polynomial and a regular chain**

One of the core commands of the RegularChains library is Triangularize. The underlying algorithm decomposes the solution set of a polynomial system into geometrically meaningful components represented by regular chains. This algorithm works by repeatedly calling a procedure, named Intersect, which computes the common zeros of a polynomial  $p$  and a regular chain  $T$ .

As the number of variables of  $p$  and  $T$ , as well as their degrees, increase, the call `Intersect(p, T)` becomes more and more computationally expensive. It was observed in (C. Chen and M. Moreno Maza, JSC 2012) that, if the input polynomial system is zero-dimensional and  $T$  is one-dimensional, then this cost can be substantially reduced. The method proposed by the authors is a probabilistic algorithm based on evaluation and interpolation techniques. This is the type of method which requires a sharp control of computing resources (in particular memory) and which is typically challenging to implement in a high-level language like Maple's language.

In this paper, we report on a successful Maple implementation of this algorithm. We take advantage of Maple's `modp2` function, developed by M. Monagan, and implemented in the C language. This function, callable within Maple's interactive loop, offers fast arithmetic for bivariate polynomials over a small prime field. Moreover, we implement the method of Chen and Moreno Maza so as to avoid unlucky specialization. Therefore, the probabilistic aspect only comes from the fact that non-generic solutions are not computed. Developing efficient modular methods for those solutions is work in progress.



**Laureano Gonzalez-Vega** (CUNEF)

**Using Maple to Study Bohemian Correlation Matrices**

Bohemian matrices are families of matrices whose entries come from a fixed discrete set of small integers. A symmetric matrix is a correlation matrix if it has ones on the diagonal and its eigenvalues are nonnegative. We will use Maple to analyze several characterizations of correlation matrices that will be used to show that the number of Bohemian Correlation Matrices over  $\{-1, 0, 1\}$  corresponds to OEIS A000110 and to solve graphically some correlation matrix completion problems.



**Sergey Yurkevich** (University of Vienna)

**The Art of Educated Algorithmic Guessing**

George Pólya described the mathematical scientific method by the short but precise premise "First guess, then prove". This mantra lies in the heart of experimental mathematics, which, while having many facets and branches, can be roughly described as a 3-step process: compute a high-order approximation of a problem, guess/conjecture a general pattern, prove the conjecture. Almost one century passed since Pólya formulated his advise and a completely new tool was invented helping to follow it even more: the computer.

Nowadays, experimental mathematics is almost impossible to imagine without computational power provided to us by recent technology. Many principles in "guessing" and "proving" were adapted and can be run completely automatized taking just fractions of seconds in time. However, experience also shows that still many techniques which are self-evident to some scientists are inaccessible or unknown to others, even though these methods could often excessively aid their research.

My talk will explain common algorithmic techniques in experimental mathematics connected to the broad topic of integer sequences. I will show how Maple's internal commands together with the packages `gfun` and `DEtools` can be used to guess and then prove highly non-trivial results. An excellent and multifaceted example is the celebrated classification of lattice paths in the quarter plane. More specific is the recent discovery and proof of the algebraicity of Zagier's sequences by Bostan, Weil and the speaker. The methods rely on very different mathematical techniques and ideas like Hermite-Padé approximation, creative telescoping, efficient solving of ordinary differential equations and the Grothendieck-Katz conjecture, however I will only briefly touch the theoretical aspects and try to concentrate on practical usage. The talk will demonstrate the above on a series of diverse examples and hopefully allow any listener an easy start in experimental mathematics using efficient algorithmic tools.

12:10 am	Networking	Lounge
12:10 am	<b>Maple Ambassador Meeting</b> (by invitation)	Theater

## Wednesday | At a Glance

Time	Event	Location
8 am – 9 am	Networking	Lounge
9 am – 10 am	<b>Keynote - Dr. Laurent Bernardin</b> Math in Changing Times	Theater
10:00 am – 10:15 am	Break	
10:15 am – 11:05 am	<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p style="text-align: center;"><b>Track 1</b> <b>Q&amp;A - Maple in Education</b></p> <p><b>Scot Gould</b> Undergraduates Learning Maple: What Sells, What Frustrates</p> <p><b>Philip Yasskin</b> MYMathApps Calculus</p> <p><b>Matteo Sacchet</b> 10 tips for successful creation of contextualized problems for secondary school students with Maple</p> <p><b>Eugenio Roanes-Lozano and Eugenio Roanes-Macias</b> A Maple-based introductory visual guide to Gröbner bases</p> </div> <div style="width: 45%; text-align: center;">  <p style="text-align: center;"><b>Track 2</b> <b>Q&amp;A - Applications of Maple</b></p> <p><b>Douglas MacDougal</b> Exploring the Mysteries of Babylonian Astronomy with Maple</p> <p><b>Maxim Sakharov, Dmitry Levando and Daniil Zaytsev</b> Endogenous demand for money, and default of a creditor</p> <p><b>Flóra Hajdu, Péter Mika, Péter Szalai and Rajmund Kuti</b> Road profile modelling for vehicle simulations using Maple</p> <p><b>Jason Whyte</b> Numerical investigation of structural minimality for structures of uncontrolled linear switching systems with Maple</p> </div> </div>	Theater
11:05 am – 11:20 am	Break	
11:20 am – 12:10 pm	<b>Discussion Panel</b> - Meet the Developers	Theater
12:10 pm – 1:00 pm	Networking	Lounge

## Wednesday | Detailed Schedule

Time	Event	Location
8:00 am	Networking	Lounge
9:00 am	<b>Keynote</b>	Theater



**Dr. Laurent Bernardin** (Maplesoft)

### Math in Changing Times

From disruptive advancements in technology to the sudden increase of remote learning and working, how we teach, learn, and do math is evolving rapidly. In this presentation, Dr. Laurent Bernardin will discuss some of the ways Maplesoft is working to ensure that everyone who touches mathematics doesn't just cope, but thrives in these changing times.

10:00 am	Break	
10:15 am	<b>Track 1: Q&amp;A - Maple in Education</b>	Theater



**Scot Gould** (Claremont McKenna, Pitzer, Scripps colleges)

### Undergraduates Learning Maple: What Sells, What Frustrates

At our undergraduate institutions, a computational applied-mathematics course is required for physics and engineering majors. This course covers nearly all the mathematics these majors would be expected to use in their undergraduate education, e.g., calculus, linear algebra, differential equations, transformations. For the past three years, the course has been taught using Maple™ and MATLAB™. At the end of the course, in addition to testing their ability to solve the problems using the software systems, students were asked to state features and attributes of the software systems they felt were most useful and increased their engagement in the material, and which they found frustrating to learn and implement. ( $N > 60$ .)

From the assessments and surveys, some major features or attributes of Maple students enjoy learning and using are:

- 2D-math including generating atomic variables and vectors arrows over variables;
- clickable choices in the palettes over function names;
- documentation features including non-executable math.
- Some major features or attributes of Maple students find frustrating to learn and/or less useful are:
  - traditional Maple coding and syntax beyond the use of parentheses, brackets and braces;
  - interpreting error statements returned by Maple;
  - the Context Panel.

In this talk, I will discuss these features and attributes and provide a primer for “selling” Maple to undergraduates. I will provide suggestions for protocols to solve math problems which will help minimize frustration. Finally, I provide some suggestions for future versions of Maple.



**Philip Yasskin** (Texas A&M University)

### MYMathApps Calculus

I am writing an online Calculus text called MYMathApps Calculus. You can see a sample at

<https://mymathapps.com/mymacalc-sample/>

The text is highly interactive and visual. Nearly all of the graphics have been made with Maple, both 2D and 3D, static and animated. The use of plots and animated plots helps students understand concepts such as

- the definitions of a derivative as the limit of slopes of secant lines, an integral as limits of Riemann sums, partial derivatives as slopes of traces, curvature and torsion, tangential and normal acceleration, divergence and curl, multiple integrals, curvilinear coordinates and Jacobians.
- the proofs of the triangle inequality, the mean value theorem and formulas for applications of integrals.
- plotting functions, polar curves, and parametric curves and surfaces.
- solving applied problems involving linear approximation, related rates, max/min, area, arc length, surface area, volumes by slicing, volumes of revolution, work, mixing problems, geometric series, Taylor series, directional derivatives, Lagrange multipliers, expansion and circulation.
- how to use the right hand rule in Green's, Stokes' and Gauss' theorems.

### **Matteo Sacchet (Università degli Studi di Torino)**

#### **10 tips for successful creation of contextualized problems for secondary school students with Maple**

Students at all levels of schooling in all countries of the world need to practice mathematical problem solving to develop competencies that they will apply in real life scenarios. On the other hand with respect to solving, problem posing refers to both the generation of new problems and the re-formulation of given problems. Teaching mathematics from a problem-posing and problem-solving perspective entails more than solving non routine problems or typical textbook types of problems. It is a way for students to exercise in all aspects of problem solving: exploring, conjecturing, examining, testing, generalizing. Tasks should be accessible and extend students' knowledge. Even students should formulate problems from given situations and create new problems by modifying the conditions of a given problem. The quality of problems submitted to students is an issue that needs to be carefully considered.

This work presents different ways to apply good practices when designing a problem-solving activity with students. It is based on the experience of the author in creating problems for Digital Math Training, a project whose aim is to develop and strengthen Mathematics and Computer Science skills through problem solving activities using the ACE Maple. After an initial training in the laboratories of the schools, 3 students per class - the most skilled or motivated ones - participate in an online training. They are asked to solve a problem every 10 days and to submit their solution. In the meanwhile, students can participate in weekly synchronous tutoring on the use of Maple and they can collaborate with their colleagues through forum discussions. Students are selected in an intermediate competition and a final one.

In this setting it is important to carefully plan and present the activity to the students, the text of the problem should be clear, concise, with little storytelling to enter into the setting of the problem. The problems should not be too theoretical, even if they could inspect specific aspects of the related theory, and need to be solved starting by simpler requests until the most difficult ones, close to the edge of students' knowledge, and making use of a calculator, in our case the Advanced Computing Environment Maple. We follow precise design principles in both the adoption of suitable practice and the use of Maple. These principles can be adapted to different situations. The paper describes all these features with examples, according to the literature and to the experience of the author.

### **Eugenio Roanes-Lozano (Universidad Complutense de Madrid) and Eugenio Roanes-Macias (Universidad Complutense de Madrid)**

#### **A Maple-based introductory visual guide to Gröbner bases**

In 1975 the Consejo Superior de Investigaciones Científicas (the main Spanish institution for scientific research) published a monograph by the second author (by the way, father and Ph.D. advisor of the first author) whose title could be translated as Geometric Interpretation of Ideal Theory (nowadays Ideal Theory is not normally used, in favour of Commutative Algebra). It somehow illustrated the geometric ideas underlying the basics of the classic books of the period (Atiyah-Macdonald, Northcot, Zariski-Samuel,...), and was a success: although written in Spanish, the edition was sold out. Of course there are much more modern books on ideals and varieties such as the famous Cox-Little-O'Shea, that illustrate the theory with images. Moreover, there are introductory works to Gröbner bases, as well as books on the topic, and articles about applications. Even a summary in English of the original Ph.D. Thesis by Bruno Buchberger is available. Nevertheless, we believe that there is a place for a visual guide to Gröbner bases, as there was a place for the monograph mentioned above. For instance, statistical packages are probably the pieces of mathematical software best known by non-mathematicians, and they are frequently used as black boxes by users with a slight knowledge of the theory behind. Meanwhile, Gröbner bases, the most common exact method behind non-linear polynomial systems (algebraic systems) solving, although incorporated to all computer algebra systems, are only known by a relatively small ratio of the members of the scientific community, most of them mathematicians. This article presents in an intuitive and visual way an illustrative selection of ideals and their Gröbner bases, together with the plots of the (real part) of their corresponding algebraic varieties, computed and plotted with Maple. A minimum amount of theoretical details is given. We believe that exact algebraic systems solving could also be used as a black box by non-mathematicians just understanding the basic ideas underlying commutative algebra and computer algebra.

 **Douglas MacDougal** (self-employed writer)

### Exploring the Mysteries of Babylonian Astronomy with Maple

Using a home computer and Maple software, we developed an educational tool to show how Babylonian astronomers likely determined the synodic returns of the outer planets with impressive accuracy over many centuries, from as far back as 400 BC.

Babylonians regarded planetary positions among the stars as omens of good or bad earthly events. Venus and Jupiter were beneficent; Mars and Saturn were malefic; Mercury was ambivalent. It was vitally important to predict when, after a whole number of cycles, a planet would return to exactly the same place among the stars in the sky. We experimented with a simple Maple computer model to see if the so-called Goal-Year periods derived over centuries of observation were at least plausibly correct. We took Mars as our case study because of its recent (October 2020) opposition and its prognostic significance to the Babylonians.

We simulated Mars's synodic cycles over 100- and 500-year periods. The Maple program searched for oppositions within a few days of being exact recurrences. The results were a success. The program: (1) came up with the 22- and 37-synodic cycle 'Goal-Year' periods found by the Babylonian astronomers; (2) confirmed the same error as found by Claudius Ptolemy for the 37-cycle return noted in his Almagest (using ancient observations collected by Hipparchus), a useful validation of our model; (3) found, using methods scholars believe the Babylonians used, the same accepted centuries-long recurrence intervals. The plotted the results were impressive.

By tweaking parameters of the program, any student can explore what-if scenarios to gain hands-on familiarity with a subject usually buried in scholarly literature and out of reach to the lay person. The concept, computation, and graphics are simple and appealing, as they were to the editors of Sky & Telescope magazine; Mr. MacDougal's feature article on the subject will appear in the August 2021 issue of that magazine.

 **Maxim Sakharov** (Bauman Moscow State Technical University), **Dmitry Levando** (Independent Researcher) and **Daniil Zaytsev** (NRU HSE)

### Endogenous demand for money, and default of a creditor

We study a general equilibrium model of perfect competition with endogenous demand for fiat (or non-consumable) money (Shubik-Wilson, 1977), with workers, entrepreneurs, and a bank. Workers supply labor (Beker, 1971) and consume, entrepreneurs consume and organize production. There is no barter, and both agent types borrow money from a bank. The bank motivates borrowers to pay loans back with a punishment, which ex ante has an impact on demands for credits.

The model has three markets: labor, goods and credits. We study interactions of these markets using numerical simulation in Maple software. The model has 4 regimes, one of which corresponds to the classical money theory. Three other regimes have defaults as parts of an equilibrium. We demonstrate, that in all three regimes too light punishment for loans results in a pass-through of default of borrowers to the creditor.

The special feature of our model is that it allows to study interactions of real (goods and labor) markets with a nominal (credit) market. We demonstrate this with numerical simulation of demand for labor and a punishment for default.

 **Flóra Hajdu** (Széchenyi István University), **Péter Mika** (Széchenyi István University), **Péter Szalai** (Széchenyi István University) and **Rajmund Kuti** (Széchenyi István University)

### Road profile modelling for vehicle simulations using Maple

This presentation gives an overview of the possibilities of road profile modelling for vehicle simulations. Standardized and measured road profiles are examined and implemented to vehicle simulations using different packages of Maple. Road profiles are created based on their Power Spectral Density function using RandomTools package to generate random phase angles. Measured road profiles are examined with Statistics and Signal Processing package to obtain their important characteristics for simulations. Curve Fitting package is utilized to create accurate input excitation signal based on measurements and generated road profile samples. Different road profile models are compared in terms of simulation time and model complexity. Numerical simulations are carried out with different degrees of freedom heavy-duty fire trucks in order to examine the effects of road quality on the vehicles and the carried equipment. The presentation concludes with further research tasks.



**Jason Whyte (University of Melbourne)**

**Numerical investigation of structural minimality for structures of uncontrolled linear switching systems with Maple**

One path to understanding a physical system is to represent it by a model structure (collection of related models). Suppose our system is not subject to external influences, and depends on unobservable state variables ( $x$ ), and observables ( $y$ ). Then, a suitable uncontrolled, state-space model structure  $S$  is defined by relationships between  $x$  and  $y$ , involving parameters  $\theta$  in  $\Theta$ . That is, each parameter vector in parameter space  $\Theta$  is associated with a particular model in  $S$ .

Before using  $S$  for prediction, we require system observations for parameter estimation. This process aims to determine  $\theta$  values for which predictions “best” approximate the data (according to some objective function). The result is some number of estimates of the true parameter vector,  $\theta$ . Multiple parameter estimates are problematic when these cause  $S$  to produce dissimilar predictions beyond our data’s range. This can render us unable to confidently make predictions, resulting in an uninformative study.

Non-uniqueness of parameter estimates follows when  $S$  lacks the property of structural global identifiability (SGI). Fortunately, we may test  $S$  for SGI prior to data collection. The absence of SGI encourages us to rethink our experimental design or model structure.

Before testing  $S$  for SGI we should check that it is structurally minimal. If so, we cannot replace  $S$  by a structure of fewer state variables which produces the same output.

Most testing methodology is applicable to structures which employ the same equations for all time. These methods are not appropriate when, for example, a process has an abrupt change in its dynamics. For such a situation, a structure of linear switching systems (LSSs) may be suitable. The structure has a collection of linear time-invariant state-space systems, and a switching function which determines the system in effect at each instant. As such, we face a novel challenge in testing an LSS structure for SGI.

We will consider the case of an uncontrolled LSS structure of one switching event (an ULSS-1 structure). In this setting, we may approach the structural minimality problem via the Laplace transform of the output function on each time interval. Each rational function yields conditions for pole-zero cancellation. If these conditions are not satisfied for almost all  $\theta$  in  $\Theta$ , then  $S$  is structurally minimal.

Analytical approaches can be quite laborious. However, a numerical approach may quickly provide useful insights. For example, if pole-zero cancellation occurs for almost all of a sufficiently large number of parameter values sampled from  $\Theta$ , then structural minimality is possible. This result may encourage us to prove the existence of structural minimality.

We shall use Maple 2020 to conduct a numerical investigation of structural minimality for a test case ULSS-1 structure applicable to flow-cell biosensor experiments used to monitor biochemical interactions.

11:05 am	Break	
11:20 am	<b>Discussion Panel</b>	Theater

**Meet the Developers**

**Want to know more about what goes on behind the scenes at Maplesoft?** This is your opportunity ask questions of senior members of the Maplesoft R&D team. The panel will include people who are highly involved with the development of various aspects of Maple, the Maple Calculator app, and Maple Learn. Between them, this panel has many (!!) years of experience developing products for doing, learning, and teaching math. This is meant to be an interactive session, so come with lots of questions!

**Panelists**

- Laurent Bernardin, President and CEO
- Andrew Smith, VP Product Development
- Erik Postma, Manager – Mathematical Software Group
- Paulina Chin, Senior Software Architect
- Jürgen Gerhard, Senior Director – Advanced Research
- Paul DeMarco, Senior Director – Maple Development
- Karishma Punwani, Director, Product Management – Academic Market

12:10 pm	Networking	Lounge
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## Thursday | At a Glance

Time	Event	Location
8 am – 9 am	Networking	Lounge
9 am – 10 am	<b>Discussion Panel</b> Another Famous Unsolved Problem: Improving Diversity in STEM	Theater
10:00 am – 10:15 am	Break	
10:15 am – 11:05 am	<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p style="text-align: center;"><b>Track 1</b> <b>Q&amp;A – Maple in Education</b></p> <p><b>Karishma Punwani</b> Next Steps with Maple Learn and Maple Calculator</p> <p><b>Paul DeMarco</b> Creating and Sharing Interactive Web Content with Maple and Maple Learn</p> <p><b>Douglas Meade</b> Creating and Using Customized MapleCloud-based Tools for Classroom and Student Use: Differential Equations</p> <p><b>Samir Khan</b> Engage your students in their engineering analyses with Maple Flow</p> </div> <div style="width: 45%; text-align: center;">  <p style="text-align: center;"><b>Track 2</b> <b>Q&amp;A – Algorithms and Software</b></p> <p><b>Amir Hosein Sadeghimanesh and Matthew England</b> Using Cylindrical Algebraic Decomposition with Equational Constraints to study Population Dynamics</p> <p><b>Amir Hosein Sadeghimanesh and Matthew England</b> A New Collection of Cylindrical Algebraic Decomposition Algorithms in Maple</p> <p><b>Tereso del Rio and Matthew England</b> Comparing the number of real roots in randomly-generated polynomials with respect to real-world polynomials</p> <p><b>Erik Postma and Marc Moreno Maza</b> Substituting Units into Multivariate Power Series</p> </div> </div>	Theater
11:05 am – 11:20 am	Break	

11:20 am – 12:10 am	<p style="text-align: center;"><b>Track 1</b> <b>Q&amp;A – Applications of Maple</b></p> <p><b>Anastasia Mukhina</b> Digital Informational Model Developing for Well Productivity Estimation through Markov Processes Tools Application</p> <p><b>John Pais</b> An Introduction to Quantum Gates, Circuits and Algorithms Using Maple Interactive Texts</p> <p><b>Athanasios Tzemos</b> Quantum Trajectories: An Overview with Maple</p> <p><b>Stephen Roper</b> Calculating the far-field, electromagnetic radiation pattern for antenna arrays that repeat in the x, y and z directions</p>	<p style="text-align: center;"><b>Track 2</b> <b>Q&amp;A – Algorithms and Software</b></p> <p><b>Bertrand Teguia Tabugua and Wolfram Koepf</b> On the representation of non-holonomic power series</p> <p><b>Maya Chartouny, Thomas Cluzeau and Alban Quadrat</b> An algorithmic approach to the algebraic parameter estimation problem</p> <p><b>Jürgen Gerhard, Marc Moreno Maza and Linxiao Wang</b> Computing the integer hull of a polyhedral set</p>	Theater
12:10 am – 1:00 pm	<b>Art Gallery Event and Networking</b>		Lounge

## Thursday | Detailed Schedule

Time	Event	Location
8:00 am	Networking	Lounge
9:00 am	<b>Discussion Panel</b>	Theater

### Another Famous Unsolved Problem: Improving Diversity in STEM

The lack of diversity in STEM fields has long been recognized as a concern. Decades ago, the focus was on trying to improve the gender balance (with some, but still insufficient success). Today, our understanding of the scope of the problem has evolved to recognize that many more groups are also missing from our classrooms, our lecterns, and our workforce. But for most people, it's still very difficult to know what we, as individuals, can do about it, or even to recognize barriers that may be outside our own lived experiences.

For some insights into this complex issue, join our discussion panel to hear from colleagues whose experiences include in-depth study of the issues, creating and supporting programs to improve diversity, and personal experiences being a minority in the STEM world. Obviously, this is not a problem to be solved at a single discussion panel, but increased understanding is always useful, and you may even pick up some ideas you can use in your own schools and workplaces.

#### Panelists

Lancelot Gooden, Durham Tech

Veselin Jungic, Simon Fraser University

Anita Layton, University of Waterloo

Kyne Santos, TikTok's "onlinekyne"

10:00 am	Break	
10:15 am	<b>Track 1: Q&amp;A - Maple in Education</b>	Theater

#### **Karishma Punwani (Maplesoft)**

##### **Maplesoft Presentation: Next Steps with Maple Learn and Maple Calculator**

In this talk, you'll learn about how Maple Learn and Maple Calculator continue to evolve rapidly as tools for teaching and learning. Maple Learn is an online math tool offering a unique, like-paper-but-better environment that puts all the visualization, solving tools, and interactivity you want at your fingertips, while also giving you the flexibility to work through steps like you would on paper. Maple Calculator is an all-in-one calculator app that lets you solve problems, check homework, and explore graphs on your phone. Both products are designed specifically to support teaching and learning of mathematics from high school to the end of second year university, and both are free to use. During this presentation you'll have the chance to see both products in action, learn about recently introduced features such as step-by-step solutions, and even get a glimpse of what is coming next.

#### **Paul DeMarco (Maplesoft)**

##### **Maplesoft Presentation: Creating and Sharing Interactive Web Content with Maple and Maple Learn**

There are new capabilities in Maple that let you easily create custom content that can be shared through Maple Learn, allowing you to offer interactive content in an online environment that also includes visualization, solving tools, and the flexibility to work through steps like you would on paper. With Maple and your own creativity, you can create self-grading practice sheets, interactive lessons

that include free-form exercises, and even exercises that ask students to provide fully worked solutions and then checks their steps to tell them where exactly they went wrong. This presentation will provide an overview of key techniques you can apply to bring your own algorithms and custom interfaces to the web through Maple Learn.

### **Douglas Meade (University of South Carolina)**

#### **Creating and Using Customized MapleCloud-based Tools for Classroom and Student Use: Differential Equations**

Maple has many powerful commands for working with differential equations. The most common commands known to most Maple users are `dsolve` to find explicit or implicit analytic solutions or approximate numeric or series solutions and `DEplot` to produce many different graphical representations of solutions. Students (and many instructors) find the details of the syntax and nuances of the output from these commands to be challenging. For many years I would create a Maple worksheet as a script for me to use in class or for students to use on their own. This approach has a number of constraints that limit its effectiveness for instructors and for students. Now, my preferred approach is to use embedded components to create a customized user interface and to make these tools accessible to students (or anyone) on MapleCloud.

In this presentation I will share some of the resources I have created, including:

- Linear Phase Plan Analyzer
- Nonlinear Phase Plan Analyzer
- Nonhomogeneous Second-Order Linear Constant Coefficients DE Solver
- Practice Applying Method of Judicious Guessing
- Visualizing Solution to 2nd order linear IVP: explicit and phase space
- Visualizing Solution of Undamped Spring-Mass Problem with Oscillating External Force

I will also share some of the techniques and habits that I have developed that have simplified the authoring of these resources. I also hope to be able to report on what appears to be a very exciting possibility for developing additional student-focused resources: Maple Learn.

### **Samir Khan (Maplesoft)**

#### **Maplesoft Presentation: Engage your students in their engineering analyses with Maple Flow**

Maple Flow is a mathematics tool that makes it easy for engineers to brainstorm, develop, and document their design calculations. Combining a simple, freeform interface with a comprehensive math engine, Maple Flow provides a whiteboard-style environment that automatically keeps calculations live as users refine, reposition, and develop their work. A useful tool for engineers everywhere, in the engineering classroom, Maple Flow offers a clean, simple environment for examples, what-if scenarios, quick scratchpad calculations, and full assignments. This talk will include an introduction to Maple Flow, and showcase many examples from different engineering domains.

10:15 am

**Track 2: Q&A - Algorithms and Software**

Theater

### **Amir Hosein Sadeghimanesh (Coventry University) and Matthew England (Coventry University)**

#### **Using Cylindrical Algebraic Decomposition with Equational Constraints to study Population Dynamics**

A simple population model with the strong Allee effect can be described by  $\frac{dx(t)}{dt} = x(t)(1-x(t))(x(t)-b)$ , where  $x(t)$  is the population size at time  $t$  and  $b$  is the Allee threshold. This model has a simple behavior. It has three steady states, extinction, Allee threshold and carrying capacity, from which the extinction and carrying capacity are stable and the Allee threshold is unstable. However, when several populations with the Allee effect are connected, the behavior of the system can get complicated. One interesting question is to determine the relation between the parameters; Allee threshold and the dispersal rate, with the number of steady states of the system. This has been focus of many studies such as [2-4,7,8].

In [7], Cylindrical Algebraic Decomposition (CAD) with respect to the Discriminant Variety is used to decompose the two dimensional parameter space to disjoint regions where the number of steady states of the three connected populations with the Allee effect is invariant. Maple already has a package to compute the open CAD with respect to the discriminant variety called

RootFinding[Parametric]. But the Gröbner bases computation for computing the discriminant variety was not feasible on their computer. Therefore a combination of this algebraic method with a numeric idea was adopted to build an approximation of this decomposition.

In this work we show how using Equational Constraints (ECs) and EC propagation [1,5,6] we can skip the Gröbner bases computation of the discriminant variety. With the help of this approach we compute the exact decomposition of the parameter space for the three connected populations with the Allee effect. All the computations are done using Maple.

#### References

1. Matthew England, Russell Bradford, and James H. Davenport. Cylindrical algebraic decomposition with equational constraints. *Journal of Symbolic Computation*, 100:38-71, 2020.
2. Mats Gyllenberg, Jarmo Hemminki, and Toomas Tammaru. Allee effects can both conserve and create spatial heterogeneity in population densities. *J. Theor. Biol.*, 56(3):231[242], 1999.
3. Diána Knipf and Gergely Röst. Large number of endemic equilibria for disease transmission models in patchy environment. *Mathematical Biosciences*, 258:201-222, 2014.
4. Diána Knipf and Gergely Röst. Spatially heterogeneous populations with mixed negative and positive local density dependence. *Theoretical Population Biology*, 109:6-15, 2016.
5. Scott McCallum. On projection in cad-based quantifier elimination with equational constraint. In *Proceedings of the 1999 International Symposium on Symbolic and Algebraic Computation, ISSAC '99*, pages 145-149. Association for Computing Machinery, 1999.
6. Scott McCallum. On propagation of equational constraints in cad-based quantifier elimination. In *Proceedings of the 1999 International Symposium on Symbolic and Algebraic Computation, ISSAC '01*, pages 223-231. Association for Computing Machinery, 2001.
7. Gergely Röst and AmirHosein Sadeghimanesh. Exotic bifurcations in three connected populations with allee effect. *bioRxiv*, 2021.
8. Irina Vortkamp, Sebastian J. Schreiber, Alan Hastings, and Frank M. Hilker. Multiple attractors and long transients in spatially structured populations with an Allee effect. *Bulletin of Mathematical Biology*, 82(6), 2020.



**Amir Hosein Sadeghimanesh (Coventry University) and Matthew England (Coventry University)**

### **A New Collection of Cylindrical Algebraic Decomposition Algorithms in Maple**

Cylindrical Algebraic Decomposition (CAD) was introduced by Collins in the 1970s [3]. Despite its complexity being doubly exponential in the number of variables, it has been used in many applications. Several different approaches to reduce the complexity of CAD algorithms have been introduced including CAD with Equational Constraints (ECs) [5, 10], PartialCAD [4] and Non-uniform CAD (NuCAD) [1]. More recently, there has been research adapting CAD for use in Satisfiability Checking, such as [8] whose CAD supports incrementality and backtracking by constraint for use in an SMT theory solver, and NLSAT [7] which uses CAD cell creation as part of a search based algorithm.

To date, these methods are not implemented in any single platform, which hinders comparison. Maple has two existing implementations of CAD, one in RegularChains [2] which produces CADs via triangular decomposition in complex space, and the other in RootFinding[Parametric] [6] which produces open CAD of the discriminant variety. Neither implementation can easily be adapted for the algorithms above, so we have produced our own Maple package. Our initial aim is to fairly compare the different CAD theories, while a future work is to integrate our package with the SAT solver in Maple [9] to produce an SMT solver in Maple for non-linear real arithmetic.

#### References

1. Christopher W. Brown. Open non-uniform cylindrical algebraic decompositions. In *Proceedings of the 2015 ACM on International Symposium on Symbolic and Algebraic Computation, ISSAC '15*, pages 85-92. Association for Computing Machinery, 2015.
2. Changbo Chen, Marc Moreno Maza, Bican Xia, and Lu Yang. Computing cylindrical algebraic decomposition via triangular decomposition. In *Proceedings of the 2009 International Symposium on Symbolic and Algebraic Computation, ISSAC '09*, pages 95-102. ACM, 2009.
3. George E. Collins. Quantifier elimination for real closed fields by cylindrical algebraic decomposition. In *Proceedings of the 2nd GI Conference on Automata Theory and Formal Languages*, pages 134-183. Cham, 1975. Springer-Verlag.
4. George E. Collins and Hoon Hong. Partial cylindrical algebraic decomposition for quantifier elimination. *Journal of Symbolic Computation*, 12:299-328, 1991.
5. Matthew England, Russell Bradford, and James H. Davenport. Cylindrical algebraic decomposition with equational constraints. *Journal of Symbolic Computation*, 100:38-71, 2020.
6. Jürgen Gerhard, D. J. Jeffrey, and Guillaume Moroz. A package for solving parametric polynomial systems. *ACM Communications in Computer Algebra*, 43(3/4):61-72, 2010.
7. Dejan Jovanović and Leonardo de Moura. Solving non-linear arithmetic. In *Automated Reasoning: 6th International Joint Conference (IJCAR)*, volume 7364 of *Lecture Notes in Computer Science*, pages 339-354. Springer, 2012.
8. Gereon Kremer and Erika Abraham. Fully incremental cylindrical algebraic decomposition. *Journal of Symbolic Computation*, 100:11-37, 2020.
9. Hui Liang, Vijay Ganesh, Pascal Poupart, and Krzysztof Czarnecki. Learning rate based branching heuristic for SAT solvers. In N. Creignou and D. Le Berre, editors, *Theory and Applications of Satisfiability Testing - SAT 2016*, volume 9710 of *Lecture Notes in Computer Science*, pages 123-140. Springer International Publishing, 2016.
10. Scott McCallum. On projection in cad-based quantifier elimination with equational constraint. In *Proceedings of the 1999 International Symposium on Symbolic and Algebraic Computation, ISSAC '99*, pages 145-149. Association for Computing Machinery, 1999.



**Tereso del Rio (Coventry University) and Matthew England (Coventry University)**

### **Comparing the number of real roots in randomly-generated polynomials with respect to real-world polynomials**

There is an increasing interest in applying Machine Learning (ML) to algebraic procedures, but as it is known, ML requires huge amounts of data and that is a scarce resource on algebraic libraries [3]. One could use random data in order to solve this problem, however, for years applied researchers have been avoiding the use of random objects because it is believed by the community that random objects have different properties than the ones that are encountered in real-world problems. In this paper that belief will be tested.

Knowing in which ways it is legitimate to generate polynomials so that they conserve the properties of real-world polynomials would be extremely useful when generating datasets to train ML models for algebraic procedures related to polynomials, such as CAD [2]. In this paper, we focus on studying whether the distribution of the number of real roots for polynomials of a particular degree is different for polynomials generated in different ways. We consider real-world polynomials to the polynomials contained in the problems stored in online libraries of symbolic data, such as SMT-LIB. From them other groups of polynomials are generated following the next strategies:

1. Generating polynomials using Maple's function `randpoly()`.
2. Model the distribution of the sparsity and coefficients of real-world polynomials and generating polynomials following that distribution with the help of Maple's function `randpoly()`.
3. Perturbate real-world polynomials (changing coefficients or degrees, or adding and subtracting monomials).
4. Create new polynomials by combining in different ways the monomials of real-world problems.
5. Project real-world multivariate polynomials using the operations in Collins' CAD projection [1] (from every polynomials on  $m$  variables one can create  $m!$  polynomials only using the discriminant for example).

The distributions of the number of real roots on every group will be compared to that distribution in real-world polynomials. This is done in order to determine which ways of generating polynomials preserve the amount of real roots and therefore could be used in studies in which this property is a very relevant factor, increasing the size of the datasets available.

#### References

1. Collins, G.E.: Quantifier elimination for real closed fields by cylindrical algebraic decomposition. In: Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), vol. 33 LNCS, pp. 134-183. Springer Verlag (1975). [http://dx.doi.org/10.1007/3-540-07407-4\\_17](http://dx.doi.org/10.1007/3-540-07407-4_17)
2. England, M., Florescu, D.: Comparing Machine Learning Models to Choose the Variable Ordering for Cylindrical Algebraic Decomposition. In: Lecture Notes in Computer Science (including subseries Lecture Notes in Artificial Intelligence and Lecture Notes in Bioinformatics), vol. 11617 LNAI, pp. 93-108. Springer Verlag (jul 2019). [http://dx.doi.org/10.1007/978-3-030-23250-4\\_7](http://dx.doi.org/10.1007/978-3-030-23250-4_7)
3. Heinle, A., Levandovskyy, V.: The SDEval Benchmarking Toolkit. ACM Communications in Computer Algebra 49(1), 1{9 (jun 2015). <http://dx.doi.org/10.1145/2768577.2768578>



**Erik Postma (Maplesoft) and Marc Moreno Maza (University of Western Ontario)**

#### Maplesoft Presentation: Substituting Units into Multivariate Power Series

The paper will discuss substituting multivariate power series into each other: suppose we have two power series  $f(x_1, \dots, x_n)$  and  $g(y_1, \dots, y_m)$ , and we want to compute  $h := f(g(y_1, \dots, y_m), x_2, \dots, x_n)$ . It is well known how to do this if  $g$  has no constant term (" $g$  is not a unit"), but  $h$  is generally not considered to be well-defined if  $g$ 's constant term,  $g(0, \dots, 0)$ , is nonzero (" $g$  is a unit"). We introduce a sense in which it is well-defined in this case, provided that we have a closed form expression for  $f(x_1, \dots, x_n)$  that we can differentiate arbitrarily often with respect to all variables and then evaluate at  $x_1 = g(0, \dots, 0)$  and  $x_2 = \dots = x_n = 0$ .

11:05 am	Break	
11:20 am	<b>Track 1: Q&amp;A - Applications of Maple</b>	Theater



**Anastasia Mukhina (National University of Oil and Gas «Gubkin University»)**

#### Digital Informational Model Developing for Well Productivity Estimation through Markov Processes Tools Application

Digital transformation of the technological operations requires the real data processing tools and methods of the enterprise information evaluation in order to lead the planning parameters results and to give the rise of industrial quality and reliability. The offered work presents the informational model creation for the correct description and investigation of the hydrocarbons production industrial processes in oil and gas area.



**John Pais** (Ladue Horton Watkins High School)

### **An Introduction to Quantum Gates, Circuits and Algorithms Using Maple Interactive Texts**

These Maple interactive texts were developed in order to introduce quantum computing into a capstone cybersecurity course. The target audience for this course is comprised of students that have already taken Calculus III (vector calculus) and/or AP Computer Science A. This course begins with an intensive study of cryptographic protocols, digital signatures, and public key cryptosystems, including all of the following: affine, Hill, Partitioned-Hill, DES, AES, Elgamal, RSA, elliptic curve, and cryptographic hash functions. After this cryptographic introduction, these encryption techniques are applied in order to achieve security for various network and internet protocols. Particularly interesting is the Transport Layer Security (TLS) protocol which in some cases may use several of these, e.g. TLS\_ECDHE\_RSA\_AES\_128\_GCM\_SHA256. Quantum computing is now an emergent technology as evidenced by the research and applications development of such companies as Google, IBM and D-Wave. Hence, quantum computing is both currently intrinsically interesting in itself, and specifically relevant to cybersecurity, as a possible threat to the TLS protocol through the theoretical ability of a large enough quantum computer to factor large integers. Furthermore, on the positive side, quantum cryptography promises secure encryption through quantum key distribution. All of which motivates the timely inclusion of quantum computing into the curriculum of a cybersecurity course.

The introduction of quantum gates, circuits and algorithms in these Maple interactive texts follows the development in "Part I: Essential Algorithms" of the excellent book by Richard J. Lipton and Kenneth W. Regan, "Introduction to Quantum Algorithms via Linear Algebra", (2nd Edition) MIT Press 2021. Since the students in our target audience are familiar with basic linear algebra and programming, but not with the 'bra' and 'ket' notation of quantum mechanics, this approach using vectors and matrices is especially clear and accessible.



**Athanasios Tzemos** (Research Center for Astronomy and Applied Mathematics of the Academy of Athens)

### **Quantum Trajectories: An Overview with Maple**

Bohmian Quantum Mechanics (BQM) is an alternative interpretation of Quantum Mechanics, according to which the quantum particles follow deterministic trajectories dictated by the so-called Bohmian equations of motion. The nonlinear character of Bohmian equations results in the coexistence of ordered and chaotic trajectories. The RCAAM of the Academy of Athens has a long tradition in the study of order and chaos in BQM.

In the last five years all the calculations have been made with Maple and Python. In fact, Maple has been an invaluable tool for the study of the wave functions of two-dimensional and three-dimensional systems. Its powerful symbolic engine and its advanced graphical capabilities helped us in producing the Bohmian equations of motion of complex systems, calculating difficult Gaussian integrals for the estimation of quantum entanglement, as well as in depicting our results with beautiful plots and animations.

The most intensive computational work in BQM is the numerical integration of the Bohmian differential systems. This can be done with Maple up to a certain level. However its capability of translating code in other languages, provided us a quick way to pass all the heavy numerical computations in Python and Fortran.

Our analytical and numerical work resulted in a series of papers covering the study of important tasks in BQM, such as the mechanism responsible for the generation of chaos in 3-D Bohmian trajectories, the phenomenon of partial integrability and the impact of the interplay between chaos, order and entanglement in the dynamical approach of Born's Rule in BQM.

BQM could serve as the starting point for a pedagogical introduction to Dynamical Systems, since all the classical methods apply naturally in the Bohmian framework. We are going to present a summary of our results through the prism of our, more than 300, algorithms in Maple 2016 and comment on our experience with Maple in general, by highlighting its unique capabilities and suggesting further improvements for its future editions.



**Stephen Roper** (EcrossH Ltd.)

### **Calculating the far-field, electromagnetic radiation pattern for antenna arrays that repeat in the x, y and z directions**

If you are interested in experimenting with simple antenna arrays, this worksheet may prove useful. I have provided a few examples of arrays that repeat in the x, y and z directions, but it will be very easy to tweak this tool if you are more interested in circular or triangular arrays.



**Bertrand Teguia Tabugua (University of Kassel) and Wolfram Koepf (University of Kassel)**

### **On the representation of non-holonomic power series**

Holonomic functions play an essential role in Computer Algebra since they allow the application of many symbolic algorithms. Among all algorithmic attempts to find formulas for power series, the holonomic property remains the most important requirement to be satisfied by the function under consideration. The targeted functions mainly summarize that of meromorphic functions. However, expressions like  $\tan(z)$ ,  $z/(\exp(z)-1)$ ,  $\sec(z)$ , etc. are not holonomic, therefore their power series are inaccessible by non-pattern matching implementations like the current Maple `convert/FormalPowerSeries`. From the mathematical dictionaries, one can observe that most of the known closed-form formulas of non-holonomic power series involve another sequence whose evaluation linearly depends on some finite summations. In the case of  $\tan(z)$  and  $\sec(z)$ , the corresponding sequences are the Bernoulli and Euler numbers, respectively. Thus providing a symbolic approach that yields explicit representations when linear summations for power series coefficients of non-holonomic functions appear, might be seen as a step forward towards the representation of non-holonomic power series.

Following Koepf's approach which computes a holonomic differential equation of least order satisfied by a given expression by solving a linear system to determine the polynomial coefficients, we define an algorithm that proceeds similarly and finds a quadratic differential equation of least order. The resulting differential equation is converted into a recurrence equation by application of the Cauchy product formula. Finally, using enough initial values we are able to give a normal form representation to characterize several non-holonomic power series by this procedure. As a consequence of the defined normal function, it turns out that our algorithm is able to detect identities between non-holonomic expressions that were not accessible in the past. We discuss this algorithm and its implementation for Maple 2022.



**Maya Chartouny (Inria Paris), Thomas Cluzeau (XLIM, University of Limoges) and Alban Quadrat (Inria Paris)**

### **An algorithmic approach to the algebraic parameter estimation problem**

The parameter estimation problem — extendedly studied in control theory and signal processing — aims at estimating unknown constant parameters of a dynamical system or a control system through system observations that can be affected by perturbations and noises. For instance, from the observation of the sum of sinusoids perturbed by an unknown bias and noise, the parameter estimation problem aims at recovering the amplitude, frequency and the phase of each sinusoid of the sum.

In 2003, Fliess and Sira-Ramirez proposed a new approach to the parameter estimation problem, called thereafter the algebraic parameter estimation problem. Within this approach, the perturbations are supposed to follow known dynamical models but with unknown parameters, and the parameters to be estimated (e.g., system parameters, initial conditions) are obtained as closed-form solutions expressed by iterative indefinite integrals of the observations and the noises only (i.e., without using the unknown parameters of the dynamical models defining the external perturbations). This approach yields real-time estimators (contrary to the asymptotical estimators in the standard literature) and the iterative indefinite integrals naturally filter the effect of the noise.

The first goal of the talk is to give a short overview of the algebraic parameter estimation problem and to illustrate it with explicit examples (e.g. numerical differentiation, finite sums of exponentials or sinusoids). We then explain how to extend the approach to the case of signals defined by linear ordinary differential equations (ODEs) with polynomial coefficients (e.g., holonomic functions, orthogonal polynomials, truncated expansions of functions onto orthogonal bases of  $L^2$ ). Moreover, using computer algebra methods (e.g., Gröbner bases for noncommutative polynomial rings of ordinary differential operators, elimination theory), we describe a general methodology for the algebraic parameter estimation problem and show how some of the steps can be made algorithmic and implemented in Maple based on the packages `intrans`, `OreModules` and `NonA`. We finally illustrate our approach by showing how the coefficients and the initial conditions of an ODE, with polynomial coefficients and without singularity at the origin, can be explicitly expressed by means of iterative indefinite integrals of a generic solution of this ODE.



**Jürgen Gerhard** (Maplesoft), **Marc Moreno Maza** (University of Western Ontario) and **Linxiao Wang** (University of Western Ontario)

### Computing the integer hull of a polyhedral set

In this presentation, we discuss a new algorithm for computing the integer hull  $P_I$  of a rational polyhedral set  $P$ , together with its implementation in Maple. Our presentation focuses mainly on the two-dimensional case.

Consider  $P$  given by a system of linear inequalities  $AX \leq b$ , where  $A$  is an integer matrix and  $b$  is an integer vector. Instead of using a conventional cutting-plane method over the whole system, we find the integer hull of each "angular sector" individually and then combine the results in order to deduce  $P_I$ . An angular sector is given by all the facets of  $P$  intersecting at one vertex of  $P$ . Our method only computes the vertices of  $P_I$ , avoiding the manipulation of all the integer points in  $P_I$ .

This algorithm is motivated by the following observation regarding parametric polyhedral sets. If one coordinate  $b_i$  of the vector  $b$  is a parameter, then there exists an integer  $T_i$  so that, for  $b_i$  large enough, the integer hull  $P_I(b_i + T_i)$  of  $P(b_i + T_i)$  can be deduced by a simple (and predictable) transformation from the integer hull  $P_I(b_i)$  of  $P(b_i)$ . Our software presentation would illustrate this pseudo-periodic phenomenon.

12:10 am

**Art Gallery Event** and **Networking**

Lounge

Join us in the Networking Room in the Lounge to learn more about the art in the exhibition, and meet some of the artists. Make sure you visit the Art Gallery ahead of time, and vote for your favorite for the People's Choice Award.

## Friday | At a Glance

Time	Event	Location
8 am – 9 am	Networking	Lounge
9 am – 10 am	<b>Keynote - Dr. Evelyne Hubert</b> An Integral View on Dimensional Analysis: Scaling Invariants for Parameter Reductions in Dynamical Systems	Theater
10 am – 10:15 am	Break	
10:15 am – 11:05 am	<div style="display: flex; justify-content: space-between;"> <div style="width: 45%;"> <p style="text-align: center;"><b>Track 1</b></p> <p style="text-align: center;"><b>Q&amp;A – Maple in Mathematics</b></p> <p><b>Yagub Aliyev</b> The best constant inequalities and their applications</p> <p><b>Zhenbing Zeng, Yaochen Xu, Jian Lu and Liangyu Chen</b> A Machine Proof of an Inequality for the Sum of Distances between Four Points on the Unit Hemisphere using Maple Software</p> <p><b>Erica Wang</b> Stability of Differential Equations with Maple</p> <p><b>Aishat Olagunju and David J. Jeffrey</b> Exploring Pythagorean n-tuples with Maple</p> </div> <div style="width: 45%; text-align: center;">  <p style="text-align: center;"><b>Track 2</b></p> <p style="text-align: center;"><b>Q&amp;A – Maple in Mathematics</b></p> <p><b>Luis Cabral and Basilides Colunche</b> Symbolic computation of dual metrics associated with Stackel-Killing tensors on curved spaces</p> <p><b>Friedemann Groh</b> Check for singular Multiplication Matrices via Macaulay-style Formula</p> <p><b>Jean-François Hermant</b> Using Maple to Learn Elliptic Curve Cryptography</p> <p><b>Valerie McKay-Crites</b> Easy Tools for Custom Visualizations</p> </div> </div>	Theater
11:05 am – 12:10 am	Break	

<p>11:20 am – 12:10 am</p>	<p style="text-align: center;"><b>Track 1</b> <b>Q&amp;A – Maple in Mathematics</b></p> <p><b>Yuri Demjanovich</b> On Complexity of Calculation for Adaptive Splines</p> <p><b>Galmandakh Chuluunbaatar, Alexander Gusev, Ochbadrakh Chuluunbaatar, Sergue Vinitzky and Vladimir Derbov</b> Algorithm for constructing Hermite interpolation polynomials on a triangle and on a tetrahedron</p> <p><b>Oleg Antoniuk</b> An algorithm to derive a rational function, that passes through chosen points, based on expressions for divided differences.</p> <p><b>Laureano Gonzalez-Vega and Alexandre Trocado</b> Visualizing the intersection curve of two quadrics with Maple: algebraic, geometric and computational issues</p>	<p style="text-align: center;"> <b>Track 2</b> <b>Q&amp;A – Algorithms and Software</b></p> <p><b>Michael Monagan</b> Speeding up polynomial GCD, a crucial operation in Maple</p> <p><b>Diego Dominici and Veronika Pillwein</b> Computing a difference equation satisfied by the Stieltjes transform of a sequence</p> <p><b>Anthony Ripa</b> Leibniz: A Generics-Based Expression Simplifier</p>	<p>Theater</p>
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## Friday | Detailed Schedule

Time	Event	Location
8:00 am	Networking	Lounge
9:00 am	<b>Keynote</b>	Theater



**Dr. Evelyne Hubert** (INRIA Méditerranée)

### An Integral View on Dimensional Analysis: Scaling Invariants for Parameter Reductions in Dynamical Systems

Dimensional analysis, also known as parameter reduction, is a recommended practice before analyzing a dynamical system, such as a physical system or biological model. The Buckingham Pi Theorem shows how linear algebra can be used to bring out dimensionless variables, as power products of the original variables, which simplifies the analysis. One issue that arises, however, is that the powers provided by the Pi Theorem can be fractional, resulting in roots, and thus they require some care when determining the regions of positivity of the variables.

In this talk, I will present an algorithm involving scaling invariants that performs a similar transformation into dimensionless variables, but the results only involve integer powers and so are much easier to work with. I will also provide a simple rewriting algorithm, in the form of substitutions, that can be used to find the induced equations in the dimensionless variables.

This talk is based on: E. Hubert & G. Labahn. Scaling Invariants and Symmetry Reduction of Dynamical Systems. Foundations Computational Mathematics. 13:4 pages 479-516 (2013)

10:00 am	Break	
10:15 am	<b>Track 1: Q&amp;A - Maple in Mathematics</b>	Theater



**Yagub Aliyev** (ADA University)

### The best constant inequalities and their applications

In this talk a parameter involving inequality about the reciprocals of  $n > 1$  positive real numbers whose sum is 1 will be discussed. For fixed  $n$  the inequality becomes stronger as the parameter increases. So it is natural to ask what is the best possible value of the parameter for which the inequality is true. We answer the question for the cases  $n=2, 3, 4$ . For  $n=3$  this inequality has some interesting connections with the inequalities in triangle geometry, in particular Euler's inequality about the ratio of radii of circumscribed and inscribed circles of a triangle. We also study this inequality for more complicated case  $n=5$ . For  $n > 5$  we discuss some asymptotic formulae for the best constant.

There is also the special case  $n=6$  for which it is theoretically possible to find an exact value of the best constant. In view of the fact that quintic equations are not in general solvable in radicals, it is unlikely that there is a precise formula for the best constant in the cases  $n > 6$ .

Variety of methods from Mathematical Analysis (Multivariable Optimization etc.) and Higher Algebra (Symmetric Polynomials etc.) were used. Also mathematical software Maple 2021 was extensively used for verification of the results.

Special cases of the discussed inequalities were extensively discussed in problem columns of various problem solving journals [1,2]. The inequality (1) is also included in the website of Research Group in Mathematical Inequalities and Applications as an open problem since 2009 [3]. The discussed problems can be regarded as a bridge between high school, olympiad, university and research level mathematics (cf. [4]).

#### References

1. M.Y.N. Aliyev, The Best Constant For An Algebraic Inequality, J. Inequal. Pure and Appl. Math. 8(2) (2007), Article 79.
2. Y.N. Aliyev, Problem 127, La Gaceta de la RSME 12(2) (2009) 315; Solution: 13(2) (2010) 308-309.
3. The RGMIA Problem Corner - 2009: <https://rgmia.org/pc/2009/problem3-09.pdf>
4. T.P. Mitev, New inequalities between elementary symmetric polynomials, J. Ineq. Pure and Appl. Math., 4(2), (2003), Article 48.



**Zhenbing Zeng** (Department of Mathematics Shanghai University), **Yaochen Xu** (Shanghai University Department of Mathematics), **Jian Lu** (Shanghai University Department of Mathematics) and **Liangyu Chen** (Shanghai Key Lab of Trustworthy Computing, East China Normal University)

### **A Machine Proof of an Inequality for the Sum of Distances between Four Points on the Unit Hemisphere using Maple Software**

In this paper, we use the exact numerical computation in Maple to prove the following theorem: If four points are placed on the unit hemisphere, and up to any rigid transformation they are not contained in certain neighborhoods of the critical points, then the sum of distances between the four points is less than  $4+4\sqrt{2}$ . The work on the construction of the critical neighborhood has been published recently in the proceedings of ADG 2021 (the Thirteenth International Conference on Automated Deduction in Geometry), where we have also proved that the sum of distances between points contained in that neighborhoods is not greater than  $4+4\sqrt{2}$ . Combining we have proved that for any four points on a hemisphere with the unit radius, the maximal sum of distances between the four points is  $4+4\sqrt{2}$ .



**Erica Wang** (Calallen HS)

### **Stability of Differential Equations with Maple**

The stability of differential equations has been studied intensively. Due to the applications of stability in system analysis and design, this topic is still of great interest for researchers. With the advance of technology, we can now use computer algebra systems to further this study. I wrote Maple procedures to demonstrate the stability of differential equations and systems of differential equations.

A differential equation is asymptotically stable when, for any initial conditions, the solutions of the equation are bounded and eventually approach zero. I used the powerful Maple function `dsolve-numeric` to write procedures. First, many initial conditions are picked up randomly. Those initial value problems are then solved, and the solutions are graphed on the same graph. One can then discern whether the differential equation is stable or unstable. If one or more solutions are getting larger and larger, clearly the equation is unstable. We may run the procedure many times; if all the solutions approach zero, we can believe the equation is asymptotically stable. Of course, this is not a theoretical proof- it can be used as verifications or to get an idea about the stability of the equation. It is also useful for teaching and learning differential equations of system theory.

I have written two procedures. One is for stability of differential equations, and the other is for stability of systems of equations.

Since the method is based on numerical approach, the equation can be either linear or nonlinear. Many unstudied problems can be investigated with this approach.

Many examples are given.



**Aishat Olagunju** (ORCCA) and **David J. Jeffrey** (University of Western Ontario)

### **Exploring Pythagorean n-tuples with Maple**

We describe some explorations of Pythagorean n-tuples using Maple. We concentrate upon triples, quadruples and quintuples. We concentrate on Pythagorean vectors (defined in previous Maple conferences). Of particular interest is the possibility of forming orthogonal pairs using Pythagorean vectors.

10:15 am

**Track 2: Q&A - Maple in Mathematics**

Theater



**Luis Cabral** (Universidade Federal N. Tocantins) and **Basilides Colunche** (Universidade Federal N. Tocantins)

### **Symbolic computation of dual metrics associated with Stackel-Killing tensors on curved spaces**

In this work, we present a symbolic computational method, using Maple 2021, for obtaining all the components of a 2-rank Stackel-Killing tensor (SKT) on a curved space and the determination of the scalar invariants related to the dual metrics (DM) of this space.

SK tensors help determine hidden symmetries in the Hamilton Jacob equation, and they help in the integrability of some geodesic equations. They are also helpful for analyzing the separability of the Helmholtz equation in curved spaces.

In addition to these applications, a relation between DM with the SKT in curved spaces was obtained in [P. T. Seidel and L. A. Cabral, Int. J. Mod. Phys. D 25 (2016) 1650022]. Such metrics occur in the presence of hidden symmetries associated with geodesics in

these spaces. A computationally demanding problem involves determining all scalar invariants for the DM, as they contain many arbitrary constants in their association with the SKT.

The SKT components satisfy partial differential equations involving covariant and symmetrized derivatives in a curved space. Here, we will present in detail how to obtain the general solution of the system and identify the terms associated with the DM together with the various arbitrary constants.

The curvature tensor from DM and its associated derivative contractions contain many terms that prevent a feasible display even with imposed simplifications. To display the scalar invariants, which have an even more significant number of elements, we developed a computational routine to select the main constants sufficient to explore the symmetric aspects of the dual space.



### **Friedemann Groh (ISG)**

#### **Check for singular Multiplication Matrices via Macaulay-style Formula**

Many technical applications require functions written in C that compute all solutions of an algebraic system. The input values of such a function are the coefficients of the system, specified in floating point format. It then returns the corresponding zeros. Typically, they are determined numerically as eigenvalues of multiplication matrices. To simplify the problem, this function does not have to be a general solver. Its source code may differ for any given algebraic system. However, in the best case it would be automatically generated using Maple.

Often the algebraic systems are specified by sparse or toric polynomials, their coefficients are algebraically independent and of degree one. So they are determined by the set of integer exponent vectors of the contained monomials, which is referred to as the support of the sparse polynomial. The convex hull of these exponent vectors is called its Newton polytope. By mixed subdivisions of the system's support sets, the multiplication matrices, necessary to compute the zeros, can be determined via Schur complements of very regularly structured sparse multivariate Sylvester matrices. Their components are either zero or a coefficient of the given system. Nevertheless, a symbolic evaluation of the Schur complement often produces extensive polynomials. Therefore, it seems reasonable to first insert the system's coefficients as floating point numbers into the multivariate Sylvester matrices and then determine the Schur complement, which involves numerical linear algebra to invert matrices.

In most cases this process works, however, singular matrices cause floating point exception errors, which is unfeasible for applications in embedded systems. Therefore, it is useful to identify these singular cases before calculating Schur complements. To this purpose, we apply an experimental algorithm to determine a mixed subdivision of the given family of support sets, which allows us to compute the u-resultant of the sparse system via a Macaulay-style formula. By means of the auxiliary linear equation for defining u-resultants, the multiplication matrices of all variables are simultaneously obtained. Due to the particular subdivision, the u-resultant can be represented by a Poisson formula. It contains a product of smaller resultants of systems formed of the sparse polynomial's initial parts, which are determined by primitive normal vectors supporting facets of the Minkowski sum of the system's Newton polytopes. If one of these facet-resultants vanishes, coordinates of a zero diverge which imply that some components of the multiplication matrices are singular.



### **Jean-François Hermant (ESIEE-IT)**

#### **Using Maple to Learn Elliptic Curve Cryptography**

In this paper, we consider one of the fastest ECC curves, Curve25519. Curve25519 was first introduced by Daniel J. Bernstein in 2005, and is now used in many applications. The curve is  $y^2 = x^3 + 486662x^2 + x$ , a Montgomery curve, over the prime field defined by the prime number  $2^{255} - 19$ , and it uses the base point  $x = 9$ . We use Maple to write a problem in cryptography for a Cybersecurity Challenge, and to solve it. An elliptic curve is given in the problem statement, with one of its point. The first step of the challenge is to recognize the ECC curve used. There is a smart (straightforward) method to recognize Curve25519 using the symmetry property of an elliptic curve. Two points of Curve25519 are computed. The first point is P1 (x1, y1). The second point is P2 (x2, y2). Ordinates y1 and y2 are given in the problem statement. The second step of the challenge is to compute Abscissas x1 and x2, given Ordinates y1 and y2. Abscissas x1 and x2 may or may not be unique. The third step of the challenge is to use the Chinese Remainder Theorem with points P1 and P2 to decipher a file.



### **Valerie McKay-Crites (Maplesoft)**

#### **Maplesoft Presentation: Easy Tools for Custom Visualizations**

The Maple Math Suite provides flexible visualization tools that are also very easy to access. In this presentation, you'll get a closer look at some of the many plot customization options available in Maple. You will learn how to experiment with interactive tools to achieve just the look you want, and still end up with a line of plotting code if that is what you ultimately need. In the final minutes, you'll also get an introduction to some of the visualization tools in Maple Learn and Maple Calculator, and how they can be used to support both independent and classroom learning.

11:10 am	Break	
11:20 am	<b>Track 1: Q&amp;A - Maple in Mathematics</b>	Theater

 **Yuri Demjanovich (Saint-Petersburg State University)**

**On Complexity of Calculation for Adaptive Splines**

The paper discusses various methods of adaptive spline approximations for the flow of function values. It is considered an adaptive compression algorithm, which, for a priori given  $\epsilon$ , has the properties 1) the complexity of the algorithm is proportional to the length of the original flow, 2) by the piecewise linear interpolation of the compression result, it is possible to restore the original flow with an accuracy of  $\epsilon$  3) the compression result is close to optimal and has of arithmetic operations. The effectiveness of this approach is demonstrated on rapidly changing initial flows of numerical information in the digital experiment. In addition, the paper presents an exact two-sided estimate for the number of arithmetic operations for the optimal solution of the problem of compressing an informational numerical flow of length  $M$  with the possibility of recovering this flow with a predetermined accuracy. Provided that the original flow is convex, a compression algorithm is developed with an accurate two-sided estimate of the number 4) and with the possibility of recovery with a prescribed accuracy.

The obtained theoretical results are illustrated using the Maple2017.0 system (see [1]) installed on the HP 27-p251ur monoblock. The values of various functions on an a priori given grid were considered as the initial flow. As a result of numerical experiments, full compliance with theoretical conclusions has been established. The report contains tables and graphs illustrating the numerical experiment.

References

1. Maple 2017.0, Product Build IDs, Maple Build ID 1231047, Licensed to: Prof. Yuri Demyanovich, Serial Number: M4SUJR24AKMC7YDY, Permanent Licence.

 **Galmankh Chuluunbaatar (Joint Institute for Nuclear Research), Alexander Gusev (Joint Institute for Nuclear Research), Ochbadrakh Chuluunbaatar (Joint Institute for Nuclear Research, Dubna), Sergue Vinitsky (Joint Institute for Nuclear Research, Dubna) and Vladimir Derbov (The Bonch-Bruевич Saint-Petersburg State University of Telecommunications)**

**Algorithm for constructing Hermite interpolation polynomials on a triangle and on a tetrahedron**

An algorithm for constructing interpolation Hermite polynomials on a triangle and on a tetrahedron is developed. These polynomials are necessary for constructing high-order computational schemes of the finite element method and ensure the continuity of the approximate solutions with their derivatives. The Hermite interpolation polynomials on simplices and prismatic polytopes are also constructed.

 **Oleg Antoniuk (Sonion Nederland B.V.)**

**An algorithm to derive a rational function, that passes through chosen points, based on expressions for divided differences.**

This presentation relates to a rational function that passes through several chosen points: it takes  $P=N+M+1$  chosen values at  $P$  chosen values of the independent variable. Here  $N$  is the order of the polynomial in the numerator and  $M$  is the order of the polynomial in the denominator of the rational function, thus the rational function has the order  $[N/M]$ .

A usual way to derive the expression for such a rational function would be to solve a set of linear equations for  $P$  coefficients describing the numerator and the denominator polynomials.

The alternative algorithm, proposed in this presentation, is based on the expressions for divided differences. For  $M=0$  the divided differences, defined in the algorithm, are identical to divided differences of the Newton polynomial interpolation equation of the order  $N$ . In this case the rational function in question becomes exactly the Newton polynomial. For  $M>0$  the definition of the divided differences is expanded into a generalized form. In this form the expression for a divided difference depends on the order of this divided difference in a piecewise way. In this way it becomes possible to address all rational functions of the order  $[N/M]$  with  $M>0$  that pass through  $P=N+M+1$  points.

A Maple routine can be written to derive the expression for a rational function based on the proposed algorithm. The expression for the rational function, derived with such a routine, is dependent on independent variable,  $P$  chosen values of the independent variable, and a set of divided differences. This is basically an explicit expression for the rational function of the order  $[N/M]$  that takes  $N+M+1$  chosen values at  $N+M+1$  chosen values of the independent variable. The derived expression happens to have a simpler structure than the same result based on coefficients of the numerator and the denominator coming from the general solution of the system of linear equations.

Also discussions of specific cases, including numerical verifications, and remaining open questions are included in the presentation.



**Laureano Gonzalez-Vega (CUNEF) and Alexandre Trocado (Universidade Aberta)**

### **Visualizing the intersection curve of two quadrics with Maple: algebraic, geometric and computational issues**

We report the algebraic, geometric and computational issues we have faced when implementing in Maple the algorithm for analyzing and visualizing the intersection curve between two quadrics presented in [L. Gonzalez-Vega and A. Trocado: Tools for analyzing the intersection curve between two quadrics through projection and lifting. Journal of Computational and Applied Mathematics, 393, art. no. 113522, 2001]. The algorithm is based on the study of the projection of this curve on a plane (the so-called cutcurve) to perform the corresponding 3D lifting correctly. When implementing this algorithm Maple was used to determine on an efficient way the topology of the cutcurve through only solving one degree eight (at most) univariate equation and several quadratic univariate equations, intersecting two pairs of conics and computing the real roots of several degree four univariate square-free polynomials whose number (of real roots) is known in advance. Finally Maple was used also to visualize such a curve and the way this issue was considered will be described too.

11:20 am

**Track 2: Q&A - Algorithms and Software**

Theater



**Michael Monagan (Simon Fraser University)**

### **Speeding up polynomial GCD, a crucial operation in Maple**

We present and discuss a Maple implementation of a new polynomial GCD algorithm. We give timing data comparing the new algorithm with Maple's polynomial GCD algorithm.



**Diego Dominici (Johannes Kepler University Linz) and Veronika Pillwein (Johannes Kepler University Linz)**

### **Computing a difference equation satisfied by the Stieltjes transform of a sequence**

We study a class of generating functions related to the Stieltjes transform of a sequence of moments with respect to the basis of falling factorial polynomials. These objects are interesting in particular in the study of discrete orthogonal polynomials as they allow to classify the discrete measures.

A generating function is called holonomic if it satisfies a linear differential equation with polynomial coefficients and likewise a holonomic sequence satisfies a linear difference equation with polynomial coefficients. This is an equivalent statement and from the defining recurrence of a holonomic sequence, the defining holonomic differential equation of its generating function can be computed automatically and vice versa.

In this talk, we consider a similar problem: given the difference equation satisfied by a sequence, how to automatically derive the difference equation satisfied by the Stieltjes transform of this sequence. We provide several examples from the class of discrete semiclassical orthogonal polynomials. Note, that for these examples, the initial difference equation can be computed from the series representation of the polynomials using classical algorithms for holonomic functions. Thus, in combination with our new procedure, we have a fully automated way to derive the recurrence for the Stieltjes transform.



**Anthony Ripa (Stony Brook University)**

### **Leibniz: A Generics-Based Expression Simplifier**

Algebra has historically relied heavily on variables, such as  $x$ . Algebra (literally, completion) varies a quantity to complete a particular kind of math problem, such as an equation (i.e. a constraint).

As mathematics has evolved, algebra techniques (specifically variables) have been applied in cases that were not, strictly speaking, equations (i.e. not constraints). This mismatch led to anomalies (e.g., removable discontinuities), which need post hoc repair. Expression simplification is an example of such a mismatch. Calculating the slope of a tangent line with algebra techniques (specifically variables) leads to such an anomaly in the form of an indeterminate form. Variable self-division is undefined at zero.

Therefore, we introduce a new primitive called a generic (written as  $x$ ), which represents a generic quantity. Generic self-division is identically 1. Generics are forward-compatible with calculus. Finally, we introduce Leibniz: A Generics-based Expression Simplifier.

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